Single-helicity states in compressible magnetohydrodynamics simulations of the reversed-field pinch with anisotropic thermal conductivity

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The reversed-field pinch (RFP) is a toroidal machine for the confinement of plasma, where the magnetic field is mainly generated by currents flowing in the plasma. The possibility of realizing the RFP configuration in the SH state, where the dynamo effect is provided by the growth of a single unstable mode, is important for plasma confinement because it results in well conserved magnetic surfaces, which ensure better confinement properties with respect to multiple-helicity states. RFP experiments have shown a tendency of the plasma towards single-helicity for increasing plasma currents. We present compressible magnetohydrodynamics (MHD) simulations of the reversed-field pinch, including anisotropic thermal conductivity.
Compressible MHD equations

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \]

\[ \frac{\partial \rho \mathbf{V}}{\partial t} = -\nabla \cdot \left( \rho \mathbf{V}_i \mathbf{V}_j + P \delta_{ij} - \nu \sigma_{ij} \right) + \mathbf{J} \times \mathbf{B} \]

\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left( \eta \mathbf{J} - \mathbf{V} \times \mathbf{B} \right) \]

\[ \frac{\partial P}{\partial t} = -\left[ \nabla \cdot (\mathbf{V} P - \kappa \nabla T) + (\gamma - 1) P \nabla \cdot \mathbf{V} \right] + (\gamma - 1) H_p \]

\[ H_p = \eta J^2 + \nu \left[ \frac{1}{2} \sigma_{ij} \sigma_{ij} - \frac{2}{3} \nabla \cdot \mathbf{V} \right] \]

Cylindrical coordinates

\[ 0 < r < 1 \quad 0 < \theta < 2\pi \quad z = R\varphi \quad (0 < \varphi < 2\pi) \]
MHD simulations of the RFP were usually performed using a simplified model with constant density and pressure.

We solve the compressible MHD equations, including density and pressure evolution in cylindrical coordinates. We use a pseudospectral method in the periodic direction, compact differences in the radial direction and an explicit Runge-Kutta time scheme.

Isosurface of density in a SH state and in a MH state
Compressible MHD simulations with anisotropic thermal conductivity
(Onofri et al., PRL 2010)

\[ \frac{\partial P}{\partial t} = -\left[ \nabla \cdot \left( V P - \kappa_{\perp} \nabla_{\perp} T - \kappa_{\parallel} \nabla_{\parallel} T \right) + (\gamma - 1) P \nabla \cdot V \right] + (\gamma - 1) H_p \]

The thermal conductivity in a magnetized plasma is anisotropic with respect to the direction of the magnetic field and for a fusion plasma the ratio \( \frac{\kappa_{\parallel}}{\kappa_{\perp}} \) may exceed \( 10^{10} \)

Thermal conduction occurs on different time scales in the parallel and perpendicular direction, so that magnetic field lines tend to become isothermal.

In a simulation it is not possible to use a realistic value of \( \kappa_{\parallel} \) because the time step would become too small.
Multiple-time-scale analysis  
(Frieman J. Math. Phys., 1963)

**Extend the number of time variables**

\[
\frac{d\tau_0}{dt} = 1, \quad \frac{d\tau_1}{dt} = \varepsilon, \quad \frac{d\tau_2}{dt} = \varepsilon^2, \quad \cdots \quad (\varepsilon \ll 1)
\]

We separate the evolution on fast time scales from the evolution on slow time scales

\[
\frac{\partial P}{\partial \tau_0} = \nabla \cdot [\kappa \nabla T] \quad (1)
\]

\[
\frac{\partial P}{\partial \tau_1} = -[\nabla \cdot (VP - \kappa \nabla T) + (y-1)PV \cdot V] + (y-1)H_p \quad (2)
\]

At each time step we look for an asymptotic solution of (1) and use it in (2)
Temperature contours and Poincaré sections of magnetic field lines \((\nu = 10^{-3} , \eta = 10^{-4} \text{ aspect ratio } R=4)\)

**Single Helicity**

**Multiple Helicity**

The simulations produce hot structures corresponding to closed magnetic surfaces in SH states and almost flat temperature when the magnetic field is chaotic in MH states.
Radial profile of temperature in MH and SH

Poincare section of the dominant mode magnetic field

A Magnetic island and an X-point are present
In RFP experiments, the resistivity increases near the wall, where temperature is lower.

Our simulations with uniform resistivity show the formation of MH states, while a stationary SH state is found when the resistivity is radially increasing, with the same on-axis value.

\[ \eta(r) = \eta_0\left(1 + 19r^{10}\right) \]

\[ \eta_0 = 10^{-4} \]

Experiments show that QSH states are more frequent for higher plasma currents. Our simulations indicate that this behavior may be due to larger temperature gradients, which determine larger resistivity gradients.
**Uniform resistivity (MH)**

\[ \eta = 10^{-4} \]

**Nonuniform resistivity (SH)**

\[ \eta(r) = 10^{-4}(1 + 19r^{10}) \]

**Energy of the most energetic modes**

- m=1, n=-10
- m=1, n=-9
- m=1, n=-8
- m=1, n=-7
Reversal parameter \( F = \frac{B_z}{\left\langle B_z \right\rangle} \) and pinch parameter \( \Theta = \frac{B_\theta}{\left\langle B_z \right\rangle} \)

Uniform resistivity

Nonuniform resistivity

![Graphs showing the behavior of F and Θ over time for both uniform and nonuniform resistivity.]

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\( F \)

\( \Theta \)
Poincaré section of magnetic field lines of the dominant mode in the poloidal plane $z = 0$

The SH state found in the simulation with nonuniform resistivity is a single-helical axis state (SHAx), the separatrix of the magnetic island disappears and a single helical magnetic axis exists.
The resistivity profile is important in the formation of SH states in the RFP. Two simulations have been performed with the same viscosity and on-axis resistivity and they show the formation of both SH and MH states, depending on the radial resistivity profile. These results indicate that the experimental finding that the QSH state is more frequently observed for increasing currents may be due to the fact that high-current discharges produce higher temperatures in the plasma core, increasing the resistivity difference between the core and the wall.