Nonlocal transport in the Reversed Field Pinch

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Introduction

- Several heuristic models for nonlocal transport in plasmas have been developed.
- There has been little possibility of detailed comparison with experimental data.
- At high current the RFP has a single helicity state with good confinement.
- At low current the reversed field pinch has a multi-helicity state with a known spectrum of relatively stable saturated tearing modes, offering a unique possibility for a study of transport in a chaotic field.
- The magnetic field in a reversed field pinch is typically chaotic but not far above stochastic threshold, so Rechester-Rosenbluth diffusion is not operative.
- Large scale structures and streamers exist in the field structure.
- A guiding center code is used to find the statistics of test particle ion transport in the given magnetic field. Possible electric potentials are neglected.
- Nonlocal transport consisting of subdiffusion is found, which can be phenomenologically fit with diffusion plus an inward pinch.
- Probability distributions of flights across the equilibrium flux surfaces are obtained.
- A nonlocal Montroll-Weiss model equation is constructed, giving a reasonable description of the essential physics.
- Relation to fractional kinetics.
RFX group - Padova

RFX is a medium-sized fusion device, whose mission is to demonstrate the fusion capabilities of a high-current reversed-field pinch (RFP) → $R = 2m$, $a = 50cm$, plasma current $I_P \approx 0.3 \div 1.8$ MA
electron temperature $T_e \sim 0.2 - 1.2$ keV, density $n_e \sim 1 \div 9 \times 10^{19}m^{-3}$
Equilibrium RFX - Padova

\[ I_p = 600kA, \quad \langle B_\phi \rangle = .2T, \quad n_i = 4 \times 10^{19}m^{-3}, \quad E = 250eV \]

(a) Toroidal and Poloidal B

(c)-(d) Field helicity \( q \), with first two resonances, \( q = 1/7 \) and \( q = 1/8 \)

The 3D MHD nonlinear, visco-resistive cylindrical code SpeCyl computes the magnetic field for a multiple helicity saturated tearing mode state with Lundquist

\[ S = 3 \times 10^4 \quad \text{and} \quad \Theta = \frac{B_\theta(a)}{B_\phi} = 1.6. \quad .15 > q > -.03 \]
Multiple and Single-Helicity States

The RFP is characterized by a rich variety of the perturbation spectrum. (c)-(d) Single Helicity state gives good confinement, essentially neo-classical \((\text{helical equilibrium}, \text{Lorenzini et al., Nature Physics 2009})\). (a)-(b) Multiple helicity corresponds to chaos in the plasma core.
Accurate representation of modes in central plasma is achieved
Single Helicity state = good description of the tomography maps
Multi-helicity Perturbation spectrum in the low current RFX

Mode spectrum obtained with code SpeCyl, using Mirnov data
\[ \delta B(\psi_p, \theta, \zeta) = \sum_{m,n} B_{m,n}(\psi_p) \sin(m\theta - n\zeta), \quad m = 1, -10 < n < 54 \]
\[ 0.15 > q > -0.03 \]
Large scale chaos with no large islands in the RFP core, \(\text{Chirikov} = 5\)
RFX - Padova

Orbit computes magnetic field lines and particle orbits using SpeCyl output. Many $m = 1$ modes with $7 < n < 26$ in the Multiple Helicity State Poincaré - general, and following field lines only until they reach $r/a = .7$. Note that there is no chaos near the axis, field lines do not pass this domain.

The $m/n = 1/7$ is stable, but the $m/n = 1/8$ is unstable

It produces $n=8$ field excursions across the whole minor radius.
Field Characterization

The parallel correlation length $L_\parallel$ is defined by the distance along the field at which $(d/dL) < r^2 >$ first reaches zero.

Kubo Number is 1.5 for perturbation amplitudes; percolation regime

$$K = \delta B_r / B_\theta(a) \ast \lambda_\parallel / \lambda_\perp$$
** Numerical simulations using the guiding center equations **

** Evaluate steady state distributions \( n(\psi) \) by fixing source and sink **

– Choose small domain, reinsert lost particles at the center with uniform pitch
– Find \( D \) from flux of particles leaving the domain and the density gradient.

** Evolution of particle distribution initiated on single flux surface \( < \Delta r^2(t) > \)**

Gather statistics on flights, defined by sign of pitch

—initial surface, direction, distance along field and across flux surfaces, time

• Trapped particles explore the field lines only for a short distance - neoclassical
• Passing particles explore the stochastic field, long flights along the field lines
• In a fully stochastic field expect Rechester- Rosenbluth diffusion

With no perturbations, nested magnetic flux surfaces, find neoclassical diffusion

\( D \) is independent of the size of the domain- No size scaling for transport.
Guiding center simulation with pitch angle scattering

Guiding center code Orbit, follows test particle ions in toroidal geometry
Collision operator, pitch $\lambda = v_{\parallel}/v$ time step $dt$, collision frequency $\nu$

$$\lambda' = \lambda(1 - \nu dt) \pm \sqrt{(1 - \lambda^2)\nu dt}$$

Lorentz, Boozer, Kuo-Petravic

- Trapped passing boundary is position dependent.
- Poincaré recurrence time simulation gives a probability distribution of time spent in a passing orbit behaving as $t^{-1.4}$ for large $t$.
- There is no mean time spent at one sign of the pitch
Phenomenological fit to transport

Transport in toroidal configurations is usually described by splitting the particle flux in a diffusive and a convective term,

\[ \Gamma = -D \frac{dn}{dx} - v \cdot n. \]

Steady state solution with source at \( x = 0 \) and sink at \( x = \pm \Delta/2 \)

\[ n = \frac{\Gamma}{v} \left[ 1 - e^{\frac{x}{D}}(x-\Delta/2) \right], \]

In the limit of \( v \to 0 \) we have \( n = \frac{\Gamma}{D} \left( \frac{\Delta}{2} - x \right), \)
Chaos generated Pinch Effect

Steady state density in GC simulation, without and with Perturbations

- Without perturbations the observed steady state distribution is a simple pyramid.
- To fit simulation data one needs a pinch term, directed against the density gradient.
- Long distance flights make the transport pitch dependent,
- Passing particles can be carried out of the simulation domain in a single flight.

Thus the pinch magnitude depends on domain size, hence on the density gradient.
Scaling by least-squares fit to $D$ and $v$

**Size—** For Lundquist $S = 10^6$, have $D \approx 7 \text{ m}^2/\text{s}$ and $v \sim 370 \text{ m/s}$, neoclassical $D \approx 0.12 \text{ m}^2/\text{s}$. Experimental pinch magnitude is about a factor of ten smaller but it scales with the domain, and the Exp. density gradient scale is ten times longer.

**Perturbation Amplitude—** $D$ starts as neoclassical ($D \sim 0.1$) and increases with mode amplitude, $D = c(\delta B/B)^{1.5}$.

**Collisionality—** $D$ initially decreases with collisions, then becomes neoclassical Pinch $v$ goes to zero — collisions curtail the mean free path.
**Topology** – • $D/v$ is nearly independent of magnetic field topology provided field is chaotic, same as fluid-kinetic theories of pinch

**Energy** – • $D/v$ is nearly independent of particle energy

**Chirikov** – $v\Delta/D$ depends on the proximity to the stochastic threshold:  
$v\Delta/D = 7.3 \ @ \ \delta B/B = 10\%$,  
$v\Delta/D = 2.7 \ @ \ \delta B/B = 4\%$
Launching particles on one surface, Monoenergetic distribution

- Launching at a/2, uniform distribution, follow $<dr^2(t)>$, collisions $\nu \simeq 1/2T$
- Toroidal angle shows transition from ballistic flow to diffusion at 10 transits
- Expect Rechester Rosenbluth diffusion at this time, but find subdiffusion
- Particles stream along the field lines giving $d\zeta \sim t$ and $dr^2 \sim t^{0.75}$
- When the motion along the field becomes diffusive $d\zeta^2 \sim t$ and $\longrightarrow dr^2 \sim t^{0.38}$
Lévy Flight Statistics

$10^7$ Lévy flights, $T =$ toroidal transits, Flight by sign of pitch

- Launch passing particles on flux surface, find statistics of flights

![Graphs showing Levy flight statistics](image-url)
Two Component Montroll-Weiss Master equation Model

• Passing particles participate in flights, trapped particles only diffuse

• Collisional transfer between trapped and passing species.

• Trapped fraction, collisional transfer rate, gyro radius - all position dependent

\[
\begin{align*}
\frac{\partial}{\partial t} n_p(r, t) &= -\nu_b(r)n_p(r, t) + \nu a(r)n_t(r, t) - \int_0^t dt' \int_0^1 dr' n_p(r', t') \int_0^1 dr P(r, r', t, t') \\
\frac{\partial}{\partial t} n_t(r, t) &= \int_0^t dt' \int_0^1 dr' n_p(r', t') P(r, r', t, t') - \nu a(r)n_t(r, t) + \nu b(r)n_p(r, t) + \frac{1}{2} \nu \rho^2(r) \partial_r^2 n_t(r, t)
\end{align*}
\]

• No source or sink the total particle number \( \int dr [n_p + n_t] \) is conserved.

• Coefficients \( a, b \) calculated using local trapped-passing boundary

• \( P(r, r', t, t') \) obtained numerically from guiding center flight statistics
- Two component fluid model, \( n_t(r, t) \), \( n_p(r, t) \), so properties must be averages over velocity (pitch).

- Mean Trapped fraction vs radius found by time asymptotic simulation

- Mean thermal energy simulation includes drifts
The propagator is evaluated by collecting statistics of flight distances. $P(r, r', t, t')$ contains all information regarding the variation of chaos and the edges. Not translation invariant, not direction invariant.
Numerically evaluated $P(r, r', t, t')$

NO FACTORIZATION - $P(r, r', t, t') \neq P(r, r') \Phi(t, t')$

Factorization violates causality, and does not account for flights being velocity dependent. $P(r, r', t, t')$ exhibits localized streamers.
Two component Montroll equation gives subdiffusion and Pinch

- Simulation up to time with toroidal flow ballistic, not diffusive

- The pinch effect is a complex interplay between passing (Lévy flight) and trapped (diffusing) particles. It cannot be reproduced with a single species.
Two component Montroll equation gives long time $t^{37}$ behavior

blue = 10, black = 100 transits, trapped = dashed, passing = solid.
**Fractional kinetics?**

- Zaslavsky has championed the use of fractional derivatives for nonlocal transport.
- Waiting time distribution, $\psi(t - t')$, and transition probability $P(r - r')$.
- Fractional kinetics is fully determined by the two parameters that describe the space and time asymptotic properties of the stochastic processes.
- The Fractional Derivative in space and time results from Lévy processes, the super- or sub-diffusive nature of the transport $< r^2 > = t^p$ is related to the asymptotic behavior of the space and time probability distributions.
- The $\alpha$th derivative of $f(x)$ is a nonlocal integro-differential operator:
  \[
  \partial_t f = \chi a D_x^\alpha f, \quad a D_x^\alpha f = \frac{1}{\Gamma(m-\alpha)} \partial_x^m \int_a^x \frac{f(y)}{(x-y)^{\alpha+1-m}} dy
  \]
  see Diego del-Castillo-Negrete, First ITER Summer School, 2007.

- But we find that the spatial transition probability is a function of the flight time.
- There is no factorization of $P(r, r', t, t')$ into $P(r, r')\psi(t, t')$ — non causal!
- The Green's function $P(r, r', t, t')$ is not translation invariant—the Lévy tail depends on location, fractional derivative $\alpha$ must be space dependent.
- In addition the bounded domain makes treatment more complicated.
- We have trapped and passing species, and the interplay between them is essential.
- We do not know how to find $P(r, r', t, t')$ nor $\alpha$ analytically from field.
Global Simulations

- The Montroll equation provides a numerically feasible means of modelling global RFX transport, including boundaries, sources and sinks.

- Uphill transport - Lévy flights effectively give a local particle sink, since some particles can jump long distances.

- This means that transport is not always “downhill” there can be local maxima, and the relation between flux and particle density is non trivial.

- Particles occaisionally participating in long flights radially is equivalent to the presence of intermittent avalanches.

- Local properties of the field and the plasma do not determine the local transport rates.
Future work

• Scaling. Note that we have fit the transport to diffusion plus pinch. For theory studies the “diffusion” constant is defined by \( < r^2 > = Dt^p \), for whatever value of \( p \) that fits the transport. To compare with Isichenko percolation scaling for example, we need to study the scaling of this \( D \).

Parallel correlation length \( L_\parallel \) can be changed by modifying the spectrum

• global simulations for steady state.

• pulse propagation studies

• Electric field effects

• Calculate the propagator \( P(r, r', t, t') \) from properties of the field? How do characteristic streamers and remnant convective cells determine the asymptotic behavior of the propagator?
Conclusions

• The RFX provides an excellent test bed for nonlocal transport studies.

• Ion transport in the RFX is subdiffusive, and a nonlocal Montroll equation captures the full nonlocal aspects of the transport. This is similar to that found for turbulent transport across stable sheared zonal flows → strong anisotropy between $\hat{\zeta}$ and $\hat{r}$ directions [Sanchez et al PRL 101, 165001 (2008)].

• Trapped and passing species, with collisional transfer between them is essential to reproduce the pinch effect observed with Orbit.

• Full scale global simulations using a Montroll equation are possible, giving a realistic steady state description of the density profile, including sources and sinks.

• The fact that the propagator in time and space is determined using guiding center simulations in the chaotic field found in RFX means that the local properties of the field and the boundary conditions are properly accounted for.