Multifractal analysis of experimental time series - does it disclose or obscure the physics?


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When do we need complex-system methods?
- Reductionist/complexicist approaches to plasma physics
- Scaling and intermittency in turbulence
- Intermittency in the solar wind and auroral electrojet
- SOC and intermittency
- The problem of trends: example from laboratory plasmas
- Stochastic modeling of signals
- Implications and conclusions
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Stochastic modeling of signals
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Implications and conclusions
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Complex-systems methods is natural to consider when it seems difficult to describe the global behavior of a system by models reduced from first principles.
Complex-systems methods is natural to consider when it seems difficult to describe the global behavior of a system by *models reduced from first principles*.
This situation is ubiquitous outside the physical sciences. It is also the case inside the physical sciences, but we tend to select our problems from the small subset that can be handled satisfactorily by the reductionist approach.
Reductionism constrains physical science

- This situation is ubiquitous *outside* the physical sciences.
- It is also the case *inside* the physical sciences, but we tend to select our problems from the small subset that can be handled satisfactorily by the reductionist approach.
When do we need complex-system methods?

Description based on first principles

Reductionist view

DOG

CAT

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When do we need complex-system methods?

Description based on function and form

Complexicist view

HUNTER

PET
Different views on the plasma zoo

Reductionist view

Complexicist view

![Graph comparing reductionist and complexicist views of plasma physics](image-url)

- Reductionist view focuses on external drive, production, and losses, highlighting boundaries and sheaths.
- Complexicist view emphasizes interaction with external circuit, boundaries, and sheaths.

Key aspects:
- Multicomponent mass ratios
- Degree of ionization
- Magnetization
- Etc.

Multifractal analysis of experimental time series - does it disclose or obscure the physics?
Pragmatically, the complex systems approach is about getting some sense out of insufficient and often incomprehensible data.
Multifractal analysis of experimental time series - does it disclose or obscure the physics?
The language of imperfect scaling, bursting, clustering, and intermittency is the formalism of multi-fractals.

It is a hard language to learn, because there are many dialects.

For instance, there are the structure function scaling exponent $\zeta(q)$, the Hentschel-Procaccia dimension spectrum $D_q$, and the singularity spectrum $f(\alpha)$, which are all equivalent representations emphasizing different interpretations.
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The dialects of multi-fractal spectra

\[ \zeta(q) \]

\[ -4 \quad -2 \quad 0 \quad 2 \quad 4 \]

\[ q \]

\[ D_q \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

\[ f(\alpha) \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

\[ \alpha \]

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Multifractal analysis of experimental time series - does it disclose
Intermittency and multifractals are about clustering of fluctuations in bursts in space and time.

The quantities $|X(t + \delta t) - X(t)|^{1/H}$ is like a distribution of “mass” on the real line into boxes of width $\delta t$. This distribution is an approximation of a measure.
Intermittency in solar wind and auroral electrojet

Intermittency does not appear in energy spectra

The energy spectra are scale-free (power-law) over more than one decade in frequency, so unveiling the intermittency characteristics requires more refined analysis.

![Power spectrum](image.png)
The amplitude of the increments $\Delta X(t) = X(t + \delta t) - X(t)$ are (in mean) proportional to the $X(t)$. This implies that increments cannot be stationary.

But the logarithm of the signal is a stationary process.

The AE-index also appears to be intermittent.
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The AE-index also appears to be intermittent.
Let $\{X_t \mid t \geq 0\}$ be a real valued stochastic process.

Define the structure functions

$$S_q(\Delta t) = \mathbb{E}[|X_{\Delta t}|^q]$$

If $\{X_t\}$ is $H$-self similar:

$$S_q(\Delta t) = c(q) \Delta t^{Hq}$$
Structure functions

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The structure function scaling exponent $\zeta(q)$

- Define $\zeta(q)$ by the relation $S_q(\Delta t) \sim \Delta t^{\zeta(q)}$ as $\Delta t \to 0$, i.e.

$$\zeta(q) = \lim_{\Delta t \to 0} \frac{\log S_q(\Delta t)}{\log \Delta t}$$

- Then we have $\zeta(q) = Hq$ for an $H$-self-similar process.
- So, computing the structure functions appears to be a good test for self-similarity.
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For a synthetic self-similar process

Structure functions and $\zeta(q)$ for a self-similar process:

![Graphs showing structure functions $S(q,l)$ and the function $\zeta(q)$ for different values of $q$.]
Is the AE index a self-similar process?

For a self-similar process, the power spectrum is

$$S(\omega) \sim \frac{1}{\omega^{2H+1}}, \quad H \approx 0.4$$

for large frequencies (time scales $< 100$ min).

![Power spectrum graph](image)
To check for self-similarity we compute structure functions:

\[
S_q(\Delta t) = \sum_{i=1}^{N-\Delta t} (x_i - \langle x \rangle)^q
\]

where \(x_i\) is the time series data, \(\langle x \rangle\) is the mean value, and \(q\) is the order of the structure function.
But $\zeta(q)$ is not a linear function

Processes with strictly increasing and nonlinear $\zeta$-functions are called *multifractal processes*.
A sandpile-like model with increasing drive

- Monofractal in the weak drive limit and multifractal in the strong drive limit.
- Can SOC/intermittent turbulence be considered as weak/strong drive limits of the same dynamical system?
A sandpile-like model with increasing drive

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Signals with trends: an example from laboratory plasmas
Profiles of plasma potential and density
The problem of trends

Time traces of plasma potential (blue) and density (red)

Red = Electron density, Blue = Plasma potential

time (ms)
The problem of trends

Spectra of plasma density and potential

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The problem of trends

Increment PDFs on increasing scales and their flatness

Electron density

Plasma potential

Red = Electron density, Blue = Plasma potential

Kurtosis of $X(t') = \frac{\Delta X(t) - \Delta \mu}{\Delta \sigma}$

Red = Electron density, Blue = Plasma potential
The problem of trends

Structure functions and $\zeta(q)$-curve for density and potential. Can I trust that the multifractality is real?
A monofractal model signal with oscillatory trends

\[ x(t) = a_1 \sin(\omega_1 t + \sigma_1 B_1(t)) + a_2 \sin(\omega_2 t + \sigma_2 B_2(t)) \]
Structure functions and $\zeta(q)$-curve for model signal. Monofractal model yields multifractal signature.
Detrended multifractal analysis.

- On each scale, subtract fitted polynomial of a given order before performing the multifractal analysis.
- The method usually performs better than direct structure-function computation, and removes spurious multifractality.
- The method usually also performs better than wavelet-based methods.
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Detrended $\zeta(q)$-curve for model signal shows monofractality.
For experimental signal multifractality survives detrending.
Implications and conclusions

- In combination with other analyses, stochastic modeling, and knowledge about the physics of the system at hand, multifractal analysis can yield further insight into the physics of the small-scale dynamics.
- Multifractal analysis of many different avalanche models might reveal an empirical connection between SOC/intermittent dynamics and weak/strong drive.
- There are many pitfalls which will yield spurious multifractality, including outliers and trends.
- Detrended multifractal analysis seems to be a method of choice.
- For complex signals with trends, stochastic difference equations with multifractal source terms have the potential to represent realistic models, which will add physical insight.
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