Recent Developments of Integrated Data Analysis at ASDEX Upgrade

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EURATOM Association

Frascati, Mar 26-28, 2012
Multi-diagnostic profile reconstruction

- Lithium beam impact excitation spectroscopy (LiB)
- Interferometry measurements (DCN)
- Electron cyclotron emission (ECE)
  - Forward modelling of the electron cyclotron radiation transport
  - $T_e$ at optically thin plasma edge
- Thomson scattering (TS)
- Reflectometry (REF)
  - $n_e$ at plasma edge
- Equilibrium reconstructions for diagnostics mapping
1. ECE forward modelling

• Magnetic fusion: Understanding and control of plasma edge
• Large variations of plasma parameters within a very thin layer
• Reliable electron density, temperature and pressure profiles
  with high spatial and temporal resolution
• Workhorse ECE: (+) plasma core, (-) plasma edge
• ECE assumptions: local emission and black-body radiation (optically thick plasma)
• Optically thin plasma edge
  → EC emission depends on density
  → combination with data from density diagnostics
  → calculate broadened EC emission and absorption profiles
    depending on $T_e$ and $n_e$
  → solve radiation transport equation
  → forward modelling in the framework of Integrated Data Analysis
Electron cyclotron intensity

Radiation transport equation:
\[
\frac{dI_\omega(s)}{ds} = j_\omega(s) - \alpha_\omega(s) I_\omega(s)
\]

Emissivity:
\[
j_m(\omega) = \frac{e^2 \omega_m^2 c^2}{8 \pi^2 \varepsilon_0 (m-1) !^2} \left( \sin \theta \right)^2 (m-1) \left( \cos^2 \theta + 1 \right)
\]
\[
\times \int \delta \left( 1 - \beta_\parallel \cos \theta \right) \omega - \frac{\omega_m}{\gamma} \left( \beta_\perp \right)^2 f (\beta_\parallel, \beta_\perp) 2 \pi \beta_\perp d \beta_\perp d \beta_\parallel
\]

Absorption
(Kirchhoff's law for thermal equilibrium):
\[
\alpha_\omega = \frac{j(\omega)}{I_{BB}(\omega)}
\]

Black-body intensity
(in Rayleigh-Jeans approximation):
\[
I_{BB}(\omega) = \frac{\omega^2}{8 \pi^3 c^2 k_B T_e}
\]
Electron cyclotron emissivity

\[ j_{2\times}(\omega) = \eta_{2\times} \ j(n_e) \ \Phi(\omega) \]

\[ \eta_{2\times} = \frac{1}{2} + \frac{1}{8} \frac{\sin^4 \theta + \cos^2 \theta}{(\cos^2 \theta + 1)^2 \cos^2 \theta + \frac{1}{16} \sin^4 \theta} \]

\[ j(n_e) = \left( \frac{e \omega_{2\times}}{4 \pi \zeta} \right)^2 \frac{n_e}{\varepsilon_0 c} \sin^2 \theta (\cos^2 \theta + 1) \]

\[ \Phi(\omega) = \frac{\zeta^{7/2}}{\sqrt{\pi}} \int \delta \left( 1 - \beta_\parallel \cos \theta \right) \omega - \frac{\omega_{2\times}}{\gamma} \right) \exp \left( -\zeta \left( \beta_\perp^2 + \beta_\parallel^2 \right) \right) \beta_\parallel^5 d \beta_\perp d \beta_\parallel \]

Emissivity for 2\textsuperscript{nd} harmonic in X-mode

2\textsuperscript{nd} harmonic in X-mode fraction only

Total emissivity

\[ \zeta = \frac{m_e c^2}{2 k_B T_e} \]

Shape function can be integrated analytically ...
Electron cyclotron emissivity

Shape function:

\[
\Phi(\omega) = \frac{\zeta^{3/2}}{\sqrt{\pi} \omega \mu \cos^5 \theta} \left[ \exp\left(-\zeta(1-\mu \alpha)\right) \right] \left[ \left( -\frac{\sin^2 \theta}{\sqrt{\alpha}} + \frac{\eta^2}{\zeta \mu} - \frac{3\eta - 2\zeta \mu \sin^2 \theta}{\sqrt{\zeta \mu}} F\left(\sqrt{\zeta \mu \alpha}\right) \right)^\alpha \right]
\]

\[
\zeta = \frac{m_e c^2}{2 k_B T_e}
\]

\[
\mu = \left( \frac{\omega}{\omega_{2\chi}} \right)^2
\]

\[
\eta = 1 + \mu \cos^2 \theta
\]

\[
\alpha = (1 - \beta_{||,1/2} \cos \theta)^2
\]

\[
\alpha_{1/2} = (1 - \beta_{||,1/2} \cos \theta)^2
\]

\[
\beta_{||,1/2} = \frac{\mu \cos \theta \mp \sqrt{1 - \mu \sin^2 \theta}}{1 + \mu \cos^2 \theta}
\]

\[
F\left(\sqrt{\zeta \mu \alpha}\right)
\]

Dawson integral (efficient subroutines and approximations)

\[
\rightarrow \text{ fast forward model for the EC radiation transport in the plasma}
\]

S. K. Rathgeber, et al., to be published
IDA: LIB + DCN + ECE(radiation transport)

- Efficient ECE radiation transport forward modeling
- High temporal (32 μs) and spatial (5 mm) resolution of edge temperature profiles
- Combination with density diagnostics (LIB, DCN)
- Quantitative reproduction of EC emission for all frequencies resonant in between the vessel walls
- *shine-through* peak explained without needing supra-thermal electrons
- *shine-through* peak provides important information about the pedestal $T_e$ gradient
IDA: LIB + DCN + ECE (radiation transport)

- Reveals steeper pedestal $T_e$ gradients compared to conventional analysis
- Pressure profiles and gradients at plasma edge
IDA: LIB + DCN + ECE(radiation transport)

- Provides information about diagnostics alignment
  - Relative shift between $n_e$ and $T_e$ → minimum in data residues

- Sensitivity study:
  - Profile parameterization with cubic spline (number and position of knots)
  - Amplitude and alignment of electron density
  - Reflection on the vessel wall (tungsten)
  - Antenna pattern
  - Additional 1$^{st}$ O-mode contribution

S. K. Rathgeber et al., to be published; PhD thesis
2. Reflectometry forward modelling

• **Goal:** \( n_e \) profiles, plasma position control (ITER)

• **Classical analysis:** Abel inversion (O-mode)
  \( \rightarrow \) location of cutoff layer

• **Problems:**
  • Multiple analysis steps (phase of reflected wave \( \rightarrow \) group delay \( \rightarrow \) density)
  • error treatment/propagation; profile uncertainties
  • density initialization outside first cutoff layer
  • *unphysical* profiles

• **IDA**
  • Forward modelling of measured data for given density profile
  • **Benefit:**
    • **Additional data** (density initialization, complementary at pedestal top)
    • Alignment
Reflectometry forward modelling

Time delay of the reflected beam (group delay):
\[ \tau(f) = \frac{1}{2\pi} \frac{\partial \phi}{\partial f} \]

Phase of reflected beam:
\[ \phi = 4\pi f \int_{r_c(f)}^{r_i} \mu(r) dr - \frac{\pi}{2} \]

Refractive index:
\[ \mu(r) = \sqrt{1 - \frac{n(r)}{n_c(f)}} \quad ; \quad n_c(f) = \frac{4\pi^2 \epsilon_0 m_e f^2}{e^2} \]

Forward model for group delay for a given density profile:
\[ \tau(f, n(r)) = \frac{2}{c} \int_0^{\sqrt{r_c-r_i}} \frac{2x}{\sqrt{1 - \frac{n(r_c-x^2)}{n_c}}} dx \]
IDA: LIB + DCN + Reflectometry

- Only physically reasonable profiles possible (spline)
- Alignment is ok (< 5 mm)
- Modification of \( n_e \) at pedestal top
• Systematic deviance in REF residue
• Minor changes in LIB residue due to modification of $n_e$ at pedestal top
• SNR(REF) < SNR(LIB)
3. IDA and the Magnetic Equilibrium

- Combine profile diagnostics LIB, DCN, ECE, TS, REF\(^\text{(new)}\)
  \[ \rightarrow n_e \text{ and } T_e \text{ profile fits to all data at once (IDA shotfile)} \]
- Mapping on a common coordinate grid using an existing equilibrium (EQH/EQI/FPP)
- Inconsistency: Equilibrium is not evaluated with kinetic profiles from IDA
  - Position of magnetic axis, separatrix, inner flux surfaces?
  - DCN: H2-H3 vertical plasma position often seems to be wrong up to ~1cm.
  - ECE: (r,z) depends on equilibrium
  - TS: vertical system relies very much on equilibrium
  - Alignment of TS, ECE, LIB (with separatrix \(T_e\)) \(\rightarrow\) uncertainties in the equilibrium ???
- Goal: combine data from profile diagnostics with magnetic data
  for a joint estimation of profiles and the magnetic equilibrium
- Needs equilibrium code:
  - CLISTE very successful, but code too sophisticated to be adapted to the IDA code
  - New code based on the ideas (success) of CLISTE
    (P. McCarthy, L. Giannone, P. Martin, K. Lackner, S. Gori)
  - Extra: Parallel Grad-Shafranov solver (R. Preuss, M. Rampp, K. Hallatschek, L. Giannone)
Grad-Shafranov solver

Grad-Shafranov equation: Ideal magnetohydrodynamic equilibrium for poloidal flux function $\Psi$ for axisymmetric geometry

\[
\left( R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2} \right) \Psi = -(2\pi)^2 \mu_0 \left( R^2 P' + \mu_0 FF' \right)
\]

1) Grid $M \times N$ (typically: radially 65 x vertically 128)

2) (a) Garchinger Equilibrium Code (GEC: Lackner et al, 1976) based on cyclic reduction
   (b) Garchinger Parallel Equilibrium Code (GPEC: Preuss et al, 2012)

3) CLISTE “Fast Mode”: solve $\Psi$ individually for $N_p + N_F$ basis functions $\pi$ and $\Phi$ (cubic spline (CLISTE), Bernstein polynomials (Giannone), Fourier-Bessel polynom.) for $P'$ and $FF'$


4) SOL: $P'$ and $FF' \neq 0$

5) Linear regression to data ($B_{pol}$, $D_{psi}$, $I_{ext}$, pressure profile, ...)

\[
\left( c, d \right) \rightarrow \Psi
\]
IDE: data and residues

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$B_{\text{pol}}$ measured

$B_{\text{pol}}$ fitted

$B_{\text{pol}}$ residue

Residues in the order of $\sim\text{mT}$

$D_{\psi}$ measured

$D_{\psi}$ fitted

$D_{\psi}$ residue
IDE: external currents and residues

I_{ext} measured and fitted:

V1o, V1u, V2o, V2u, V3o, V3u,
OH1 = OH3o = OH3u = OH,
OH2o = OH + dOH2s,
OH2u = OH + dOH2s + dOH2u,
Coiu, Coiu,
Ipslon, Ipslun
Comparison EQH/IDE: pressure and poloidal current

Grad-Shafranov: \( p' \)

Pressure profile and IDA pressure constraints
\( (T_i = T_e, Z_{\text{eff}} = 1.5, \text{uncertainty 50\%}) \)
- Center: Pressure gradient \( \rightarrow p' \)
- Edge: \( p \) and \( p' \)

Net poloidal plasma current
(wo external currents)
- Center: Pressure constraints reduce \( l_{\text{pol,net}} \)
- Edge: Opposite direction
Comparison EQH/IDE: Temperature and density

#25764, 2.0s
IDA(EQH)
IDA(IDE)

edge

core

~5cm

~5mm

$T_\rho$ [keV]

$n_e$ [m$^{-3}$]

$\rho_{pol}$

$\rho_{pol}$
SUMMARY: IDA at ASDEX Upgrade

- **ECE forward modelling**
  - Solving the radiation transport equation
  - Combination with density diagnostics (LIB, DCN)
  - *shine-through* peak (no supra-thermal electrons, pedestal $T_e$ gradient)
  - Diagnostic alignment
  - Sensitivity study

- **Reflectometry forward modelling**
  - Group delay for given density profile
  - Error propagation $\rightarrow$ uncertainty of profiles
  - Redundant information $\rightarrow$ resolve data inconsistencies
  - Complementary information $\rightarrow$ pedestal-top density

- **IDA and magnetic equilibrium**
  - New equilibrium code with parallel Grad-Shafranov solver
  - Consistency of profiles and equilibrium
  - Reduction of ill-posedness
  - Edge current distributions