3D LH GRILL COUPLING AND EFFICIET FULL WAVE CODE

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The lower hybrid waves are very important for heating and current drive in present-day tokamaks.The phased arrays of rectangular waveguides are frequently used as slow down structure. Simple theory of parallel plate waveguide coupling to the plasma slab with the linear density profile is often sufficient. However, it neglects coupling fast to slow wave in inaccessibility region and cannot handle the effect of poloidal field (magnetic shear and oblique magnetic field with respect of the long wall of waveguide). These drawbacks were overcome by 3D grill theory assuming the rectangular waveguides and 1D full wave solution in plasma. However, this method is rarely used for large structures as it requires the evaluation of a very large number of 2D infinite integrals in k-space for coupling elements . We developed a method based on high order Gaussian quadrature combined with 2D B-splines of integrands in space to overcome this drawback. At the same time we adopt an efficient procedure to accurately determine the eigenmodes excited by LH grill in inaccessible region and clear up the role of collisions in this context .

As a an example of this method we test several grills for COMPASS tokamak.

Review of the LH grill theory

z-axis||B –toroidal direction, y-axis in poloidal direction

Field in the waveguides – superposition of incident and reflected TE_{10} modes
+ higher evanescent TE and TM modes

Field in the vacuum gap – superposition of incident and reflected plane waves

$$E_\alpha = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dN_y dN_z \mathrm{e}^{\mathrm{i}(N_y y' + N_z z')}\left(A_\alpha \mathrm{e}^{\mathrm{i}(N_x x')} + R_\alpha \mathrm{e}^{-\mathrm{i}(N_x x')}\right), \alpha = y, z$$

Amplitudes of waves reflected from plasma – $R_\alpha(N_y, N_z) = \sum_\beta A_\beta R_{\alpha\beta}$

Matrix of reflection coefficients $R_{\alpha\beta}$ – from the electric field on the plasma boundary determined by full wave solution

$$R_{yy} = E_y(x_p) - E_y^{inc}, \quad R_{zy} = E_z(x_p), \quad E_y^{inc}=1, E_z^{inc}=0$$

$$R_{zz} = E_z(x_p) - E_z^{inc}, \quad R_{yz} = E_y(x_p), \quad E_y^{inc}=0, E_z^{inc}=1$$

Without shear – symmetry rules

$$R_{\alpha\alpha}(N_y, -N_z) = R_{\alpha\alpha}(N_y, N_z),$$

$$R_{\alpha\beta}(N_y, -N_z) = -R_{\alpha\beta}(N_y, N_z), \quad \alpha \neq \beta$$

Field in the plasma

We use full wave solver from **AMR** code

Propagation of the electromagnetic waves in the cold inhomogeneous magnetized collisional plasma is solved by finite elements method with adaptive mesh of nodes and also adaptive length of the integration interval.

Two 2nd order differential equations for $E_y^{\text{plasma}}, E_z^{\text{plasma}}$ corresponding to the electric field of the wave are solved.

$$iN_y E'_x + \varepsilon_{xy} E_x - E''_y + \left(N_z^2 - \varepsilon_{yy}\right) E_y - \left(N_y N_z + \varepsilon_{yz}\right) E_z = 0,$$

$$iN_z E'_x + \varepsilon_{xz} E_x - E''_z - \left(N_y N_z + \varepsilon_{yz}\right) E_y + \left(N_y^2 - \varepsilon_{zz}\right) E_z = 0,$$

$$E_x = \frac{1}{N_y^2 + N_z^2 - \varepsilon_{xx}} \left(-iN_y E'_y - iN_z E'_z + \varepsilon_{xy} E_y + \varepsilon_{xz} E_z\right).$$

Boundary condition at the plasma surface corresponds to the continuity of the tangential components of the electric and the magnetic fields of the wave.

Boundary condition at the plasma depth corresponds to the outgoing or decaying waves

The high relative precision of (10^{-4}) of the electric fields in plasma is readily reached.

Plasma model accepts the full magnetic configuration obtained e.g. from EFIT and arbitrary profiles of density and temperature (for determination of realistic profile of the collision frequency), so we can consider coupling of fast and slow waves in an inaccessibility region, oblique magnetic field at the grill mouth, poloidal magnetic field, magnetic shear.

Plasma Reflection Coefficient in (N_y, N_z) – space

illustrating figures are for COMPASS 8 waveguide grill (1.3GHz) phased at $\Delta\varphi=\pi/2$

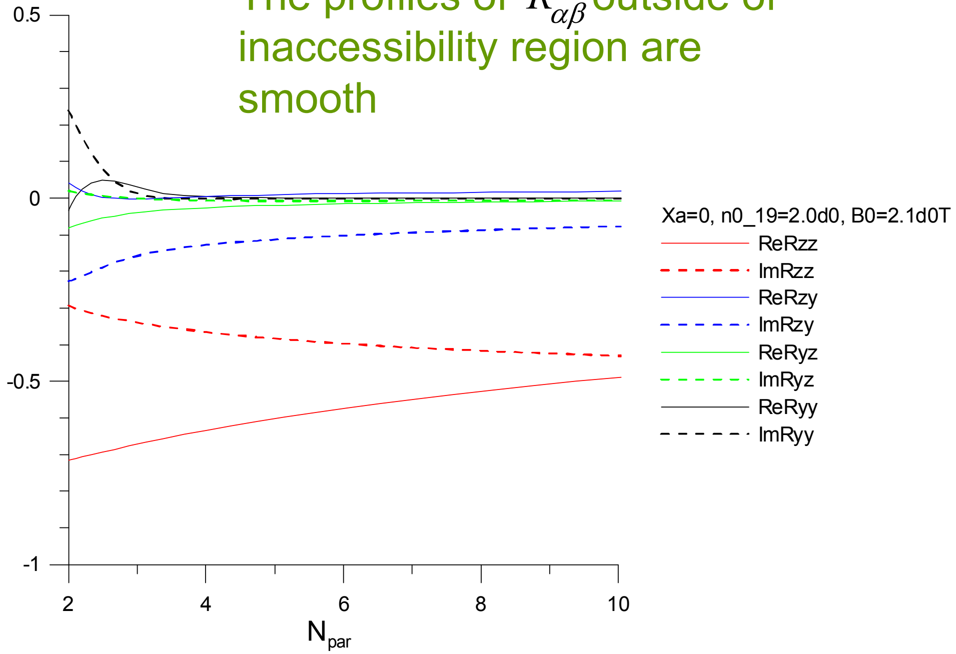
Accessibility

For $N_{\parallel}^{access} < \omega_{pe}(R_0)/\omega_{ce} + \sqrt{1-\omega_{pi}^2(R_0)/\omega^2+\omega_{pe}^2(R_0)/\omega_{ce}^2(R_0)}$ fast and slow wave are mutually converted at some distance from the plasma boundary.

Eigenmodes

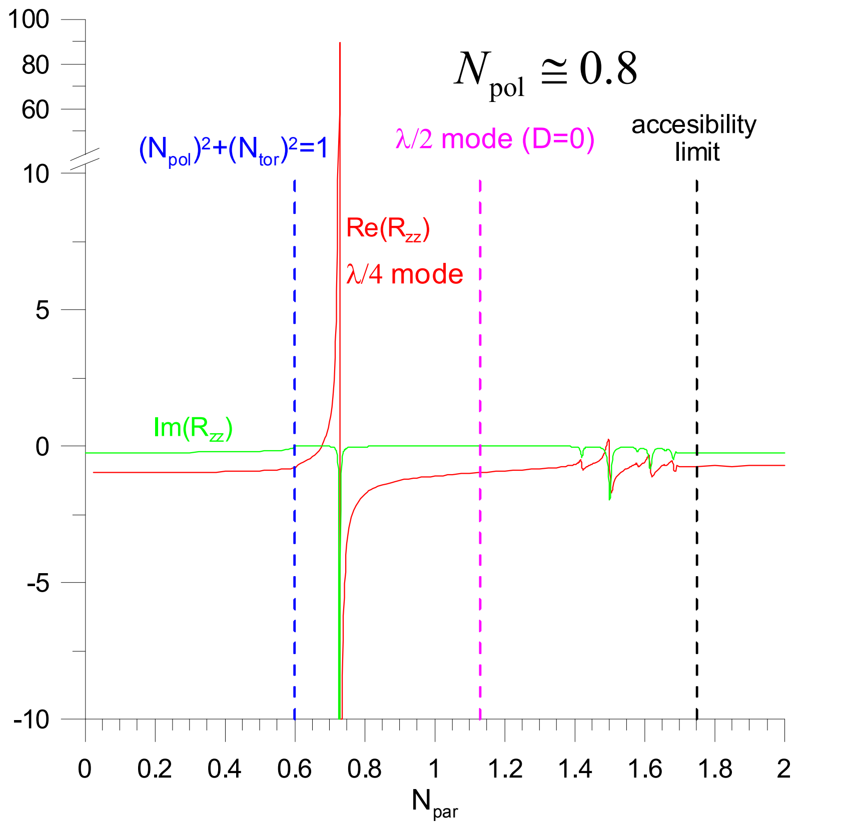
Eigenmodes emerge if integer number of quarters of wavelength fits to this distance.

The profiles of $R_{\alpha\beta}$ outside of inaccessibility region are smooth



Reflection coefficient

Real and imaginary parts of R_{zz} look like the compliance of forced vibrations of damped harmonic oscillator .



Continuity of the electric field in the plane of grill mouth

Fourier analyzing the tangential components of the wave electric field we obtain amplitudes of waves incident on plasma

$$A_\alpha = Q_\alpha \tilde{R}_{\beta\beta} - Q_\beta \tilde{R}_{\alpha\beta}, \quad \tilde{R}_{\alpha\beta} = \frac{\delta_{\alpha\beta} + R_{\alpha\beta}}{D}, \quad D = \left(1 + R_{yy}\right)\left(1 + R_{zz}\right) - R_{yz} R_{zy}$$

Singularity

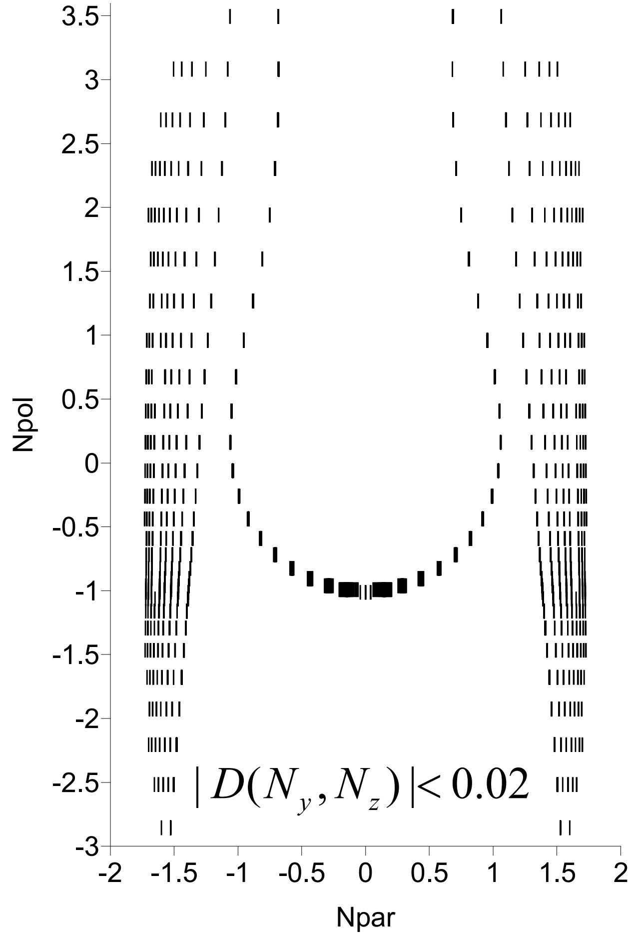
If $D(N_y, N_z) = 0$ fast and slow waves have the same polarization on the plasma surface and the solutions are linearly dependent

System determinant D=0 only if collisional frequency $\nu = 0$: hence our solution are regular

Eigenmodes of plasma slab

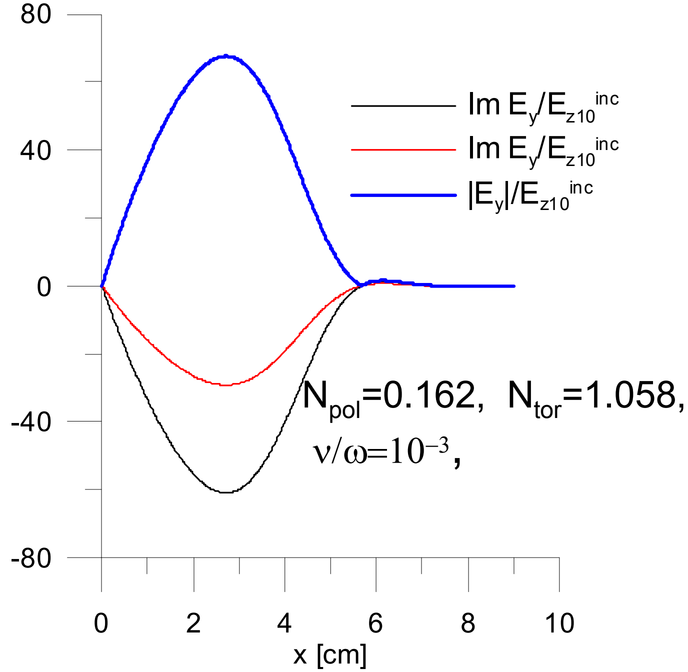
Eingenmodes with integer number of half wavelengths have node at plasma boundary so fulfill condition

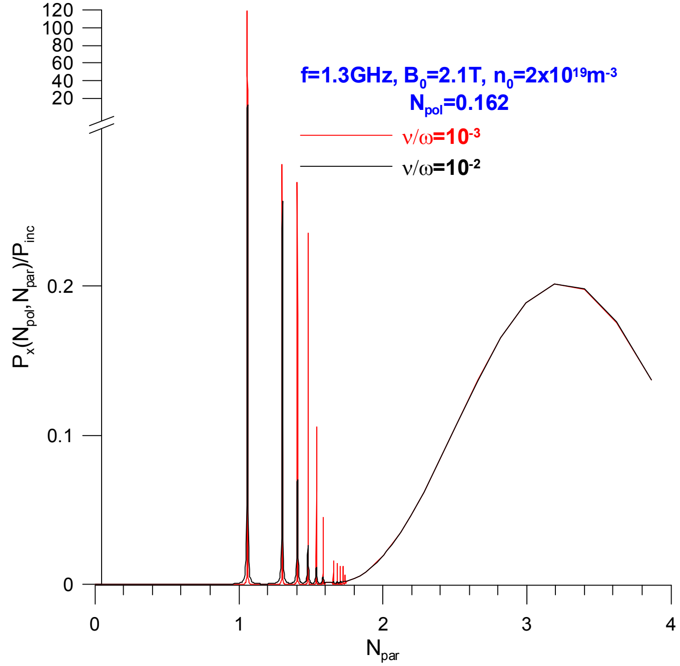
Only eigenmodes with node on the plasma surface are exited by grill

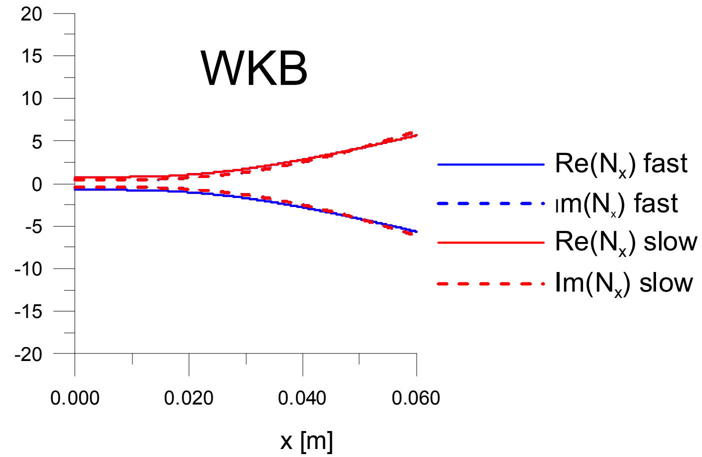


Details of eigenmodes in the inaccessibility region

Excited modes have nodes at the plasma surface. The first super mode is a pure full wave solution. The WKB approximation totally fails. The electric field is normalized to the unit amplitudes of the incident waves in the waveguides Both components E_y, E_z have the same size

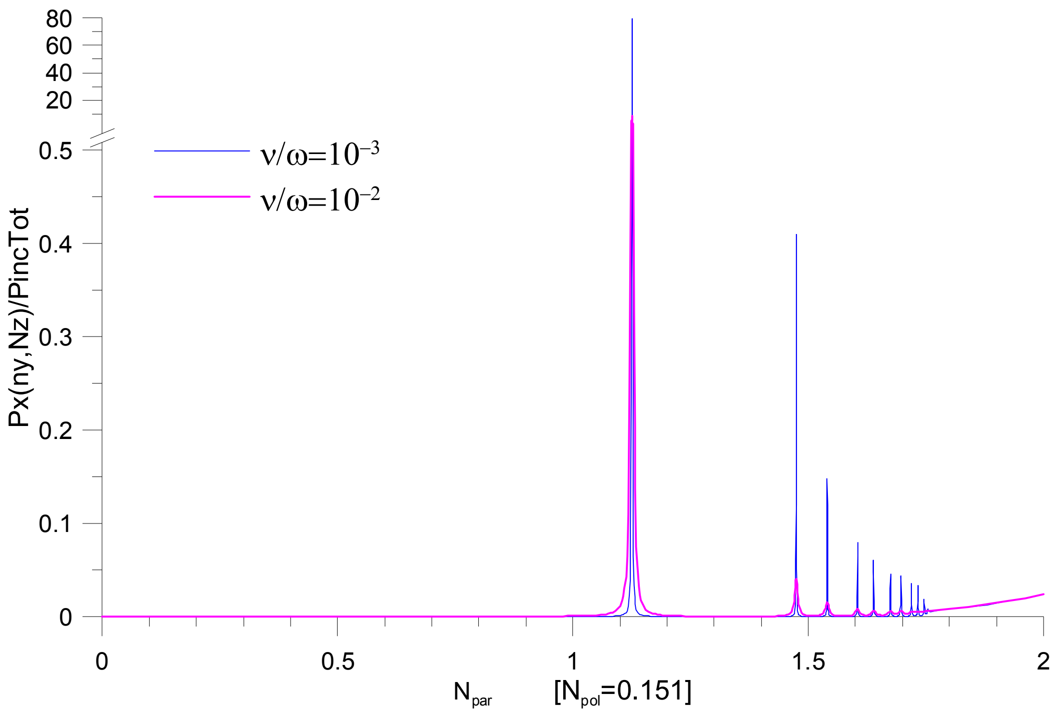


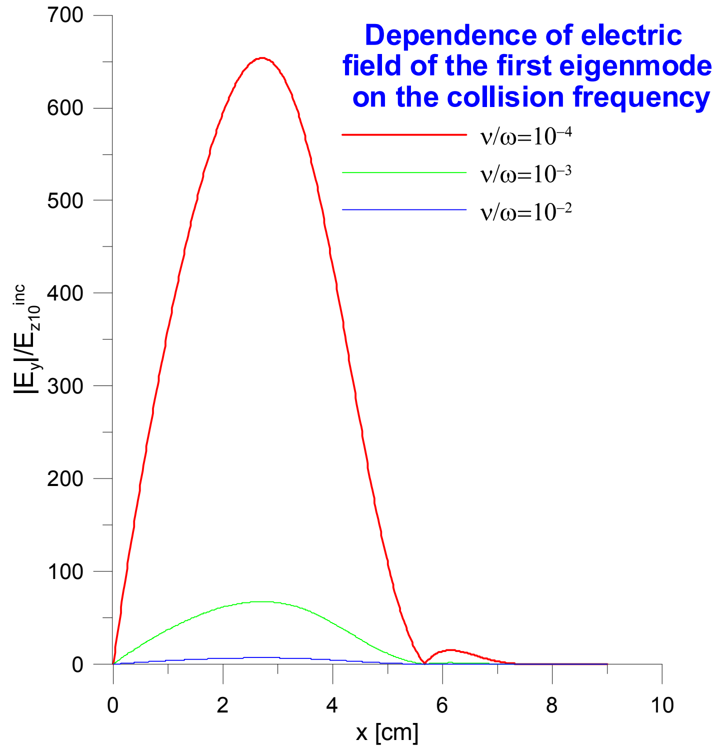




Effect of collisions

Homogeneous collision frequency is used as a model parameter to keep resonances finite
Its value does **not influence the shape of power spectrum** in the accessible part of (N_y,N_z) plane, **the power reflection coefficient nor the directivity**. However, the peaks in the inaccessible part of the spectrum are higher and narrower for lower ν.
Power transmitted to the standing eigenmodes does not depend on the collisional frequency.
The electric field of standing eigenmodes in the inaccessible region increases substantially but superposed field in x-space is independent of collisions (with exception of radial damping).
We can obtain realistic picture of grill coupling for $\nu / \omega = 10^{-2}$ where full wave integration is easy





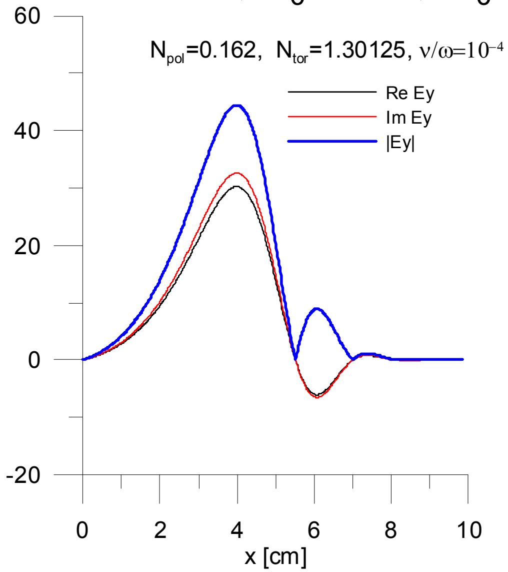
Details of higher eigenmodes

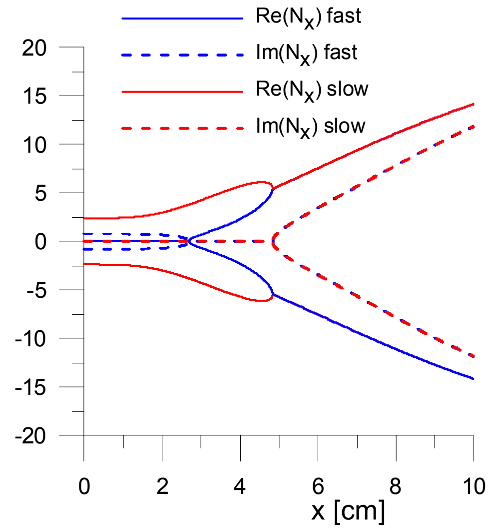
f=1.3GHz, B_0=2.1T, n_0=2x10^{19}m^{-3}, ν/ω=10^{-4}

2nd mode N_y=0.162, N_z=1.301

Full wave solutions is consistent with WKB approximation.

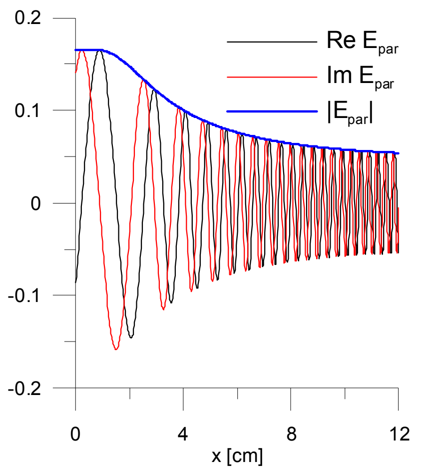
Coupling of the fast and slow wave is seen in profiles of their WKB wave vectors

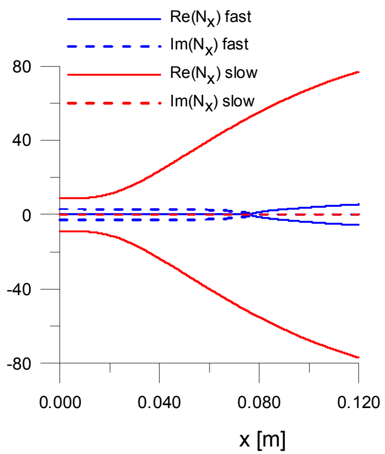


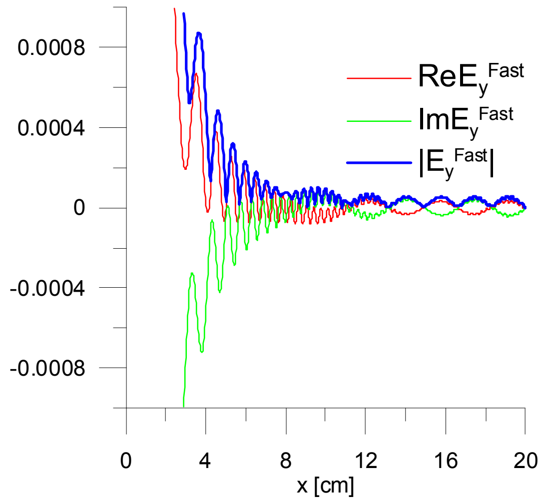


Details of propagating modes

Transmitted field is composed mainly of slow waves, but a weak tunneling of fast wave is also observable (the last figure represent the WKB decomposition of full wave solution into partial slow and fast waves
The main part of the power is radiated as the slow waves with $|N_y|$<2 and 2<N_z<5







Continuity of the magnetic field at the waveguides mouth's

The tangential components of wave magnetic fields are continuous at the throats of waveguides. The waveguide eigenfunction are orthogonal, yielding the required $(2N_{\mathrm{mod}}M_{\mathrm{mod}}+M_{\mathrm{mod}}+N_{\mathrm{mod}})N_w$ equations for unknown B^H_{mnp} and B^E_{mnp}

The coefficients in these equation are the **coupling admittances**

$$K^{zz}(m,n,p,k,j,q)=$$

$$(4\pi^2)\int_{-\infty}^{\infty}dN_y J_m^S(N_y)\left(J_k^C(N_y)\right)^*\int_0^{\infty}dN_z I_{zz}^{plasma}(N_y,N_z)\left(I_{np}^C(N_z)\left(\mathrm{I}_{jq}^S(N_z)\right)^*-\left(I_{np}^C(N_z)\right)^*\mathrm{I}_{jq}^S(N_z)\right)$$

Plasma related part of the integrand does not depend on the grill structure and it is determined by computantioally expensive full wave solutions of wave propagation in the plasma. It is smooth function of N_y and N_z out of the inaccessibility region but it has a complicated shape within the inaccessibility region (N_z<1.75 in our case)

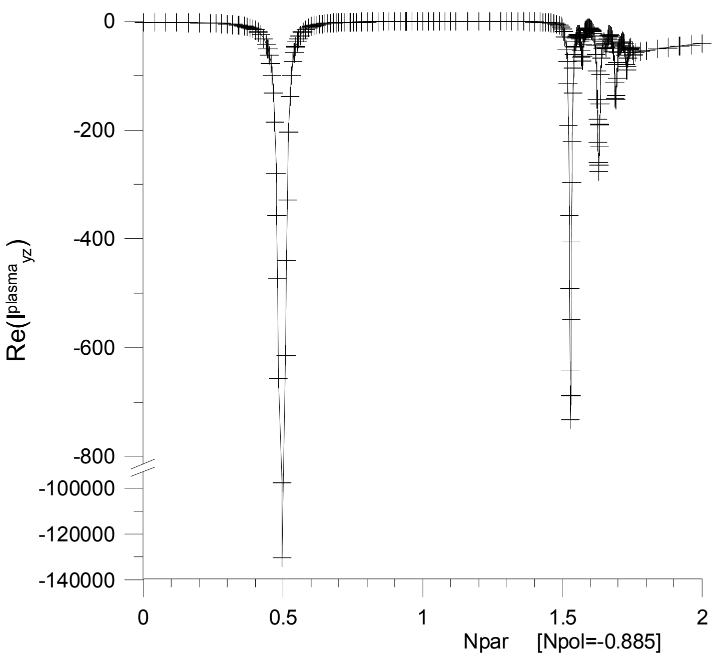
$$I_{zz}^{plasma}(N_y,N_z)=-\frac{2(1-N_z^2)\overline{R}_{yz}+N_yN_z((\overline{R}_{yy}+1)(\overline{R}_{zz}-1)-\overline{R}_{yz}\overline{R}_{zy})}{\left((\overline{R}_{yy}+1)(1+\overline{R}_{zz})-\overline{R}_{yz}\overline{R}_{zy}\right)N_x}$$

Fourier coefficients of y- and z- space dependent waveguide modes $J_m^{C,S}(N_y), I_{np}^{C,S}(N_z)$ are simple oscillating functions of N_y and N_z

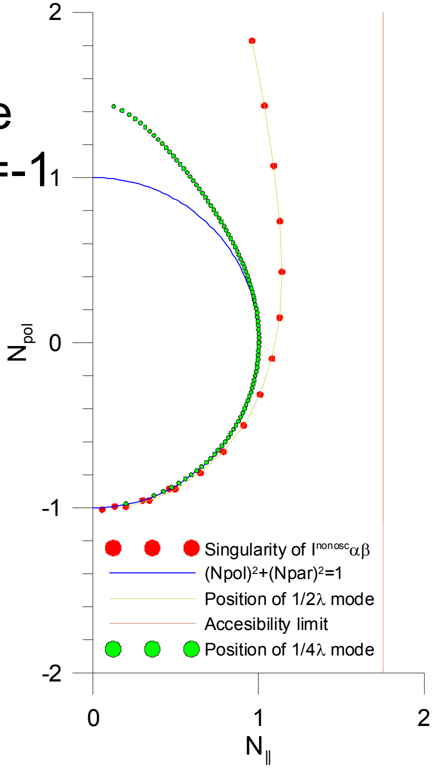
We have to evaluate enormous number of 2D integrals $K^{\alpha\beta}(m.n,p,k,j,q)$
(e.g. our 8 waveguide grill and for six toroidal and 6 poloidal modes more that 300 000)

Application of high order Gaussian rule for N_{par} > N_{acc} and trapezoidal rule for 0< N_{par} < N_{acc}

We use 128 point Gaussian rule to integrate the admittances from $N_{acc} < N_{par}^{\max}$ and trapezoidal rule for $0 < N_{par} < N_{acc}$. Than the results are integrated by two Gaussian rules over two intervals $N_{pol}^{\min} < N_{pol} < -1$ and $-1 < N_{pol} < N_{pol}^{max}$ Plasma related part of integrand is iteratively tabulated for trapezoidal rule. Only one mesh of N_{pol}, N_{par} points is used for all m,n,p,k,j,q. Symmetry rules on m,n,p,k,j,q reduce number of computed integrals hundred times.



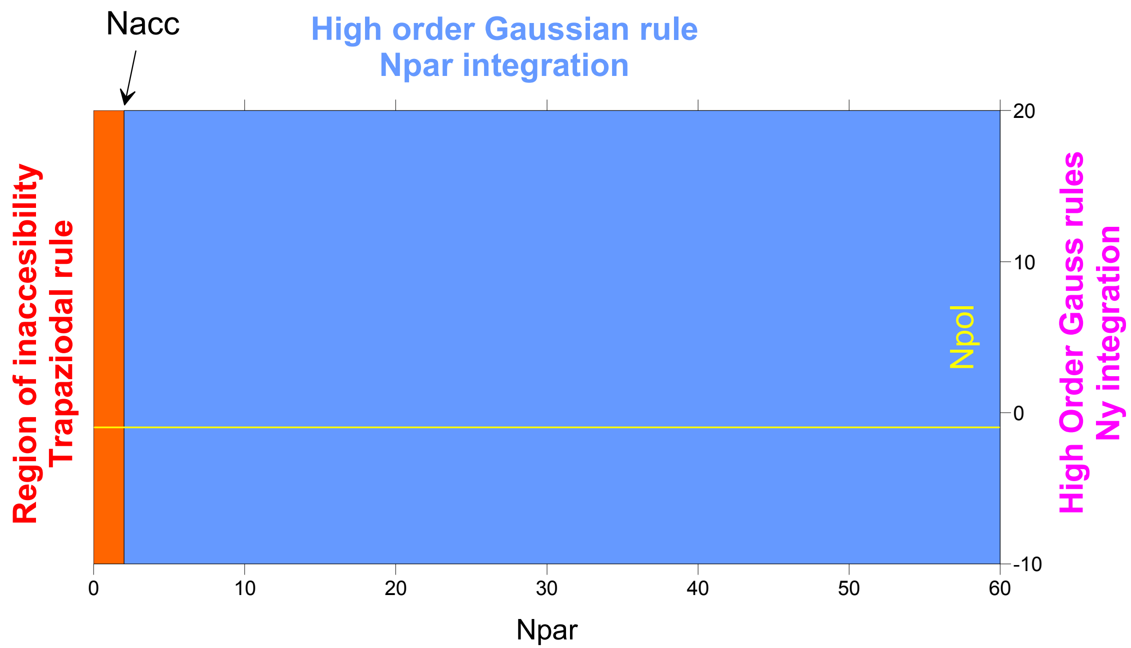
The most singular part of N_{pol}, N_{par} plane is situated near N_{pol}=-1 when several problematic features coalesce. For a finite collisional frequency ($\nu/\omega=10^{-4}$) situation can be handled



Division of (N_{pol},N_{par}) plane used in the 2D integration of coupling admittances

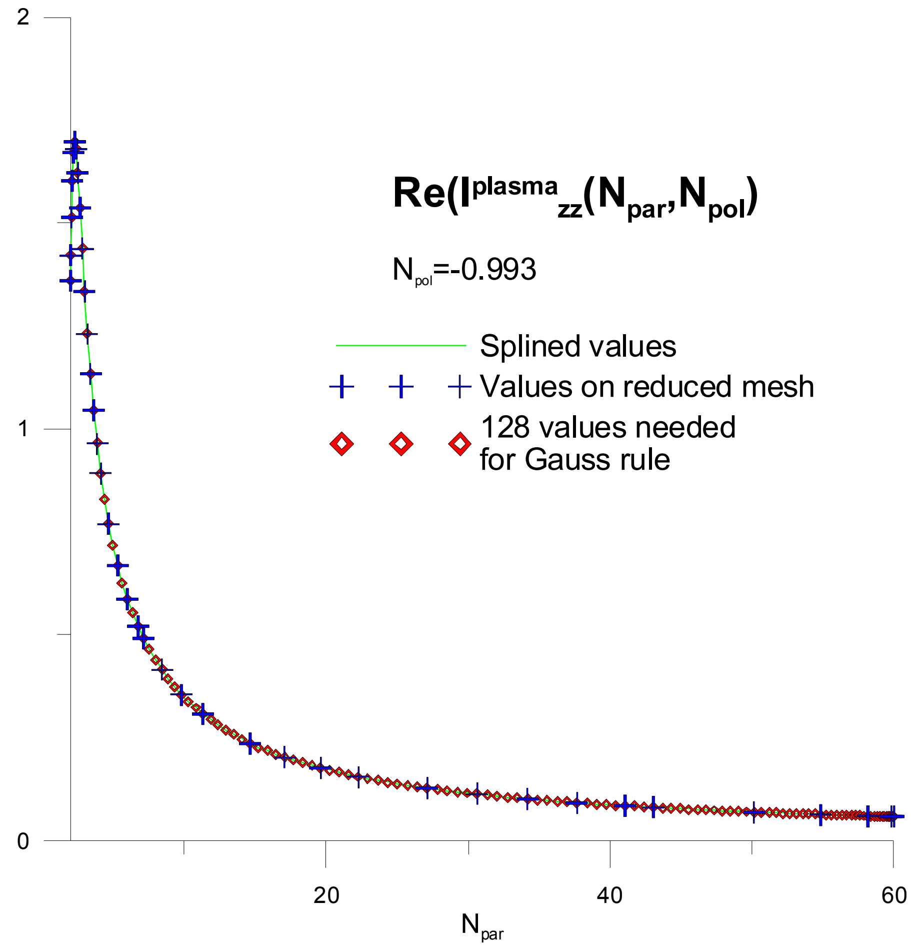
Order of Gauss rule must be adjusted along with the number of waveguides and modes

For COMPASS grill (8 waveguides) and 6X6 modes Gauss rule with 128 nodes is sufficient in N_{par} and 32 and 64 nodes are used in N_{pol} direction (below $N_{pol}^{\min} = -10$ and above $N_{pol}^{\max} = 20$ the eigenmodes are not detectable)



Number of required full wave solutions can be substantially reduced by 2D Bsplining of plasma related parts of the integrand

Plasma related parts of the integrand are usually smooth function of N_{par} in accessible region and the integrals over N_{par} are also smooth function of N_{pol} so we evaluate integrand on reduced mesh in both N_{par} and N_{pol} and remaining Gauss values are obtained by 2D Bsplines



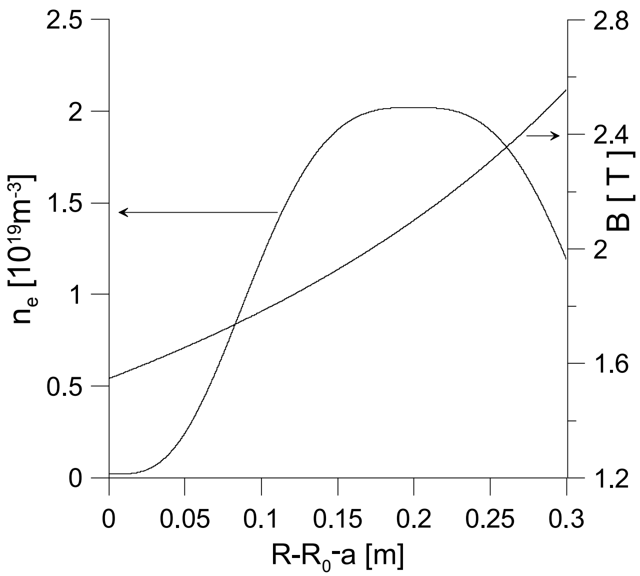
Tokamak COMPASS and its LH Grill

COMPASS is a medium size tokamak now at IPP Prague

Major radius R_{major} =0.56 m Minor radius R_{minor}=0.2 m Central magnetic field B_0<2.1 T

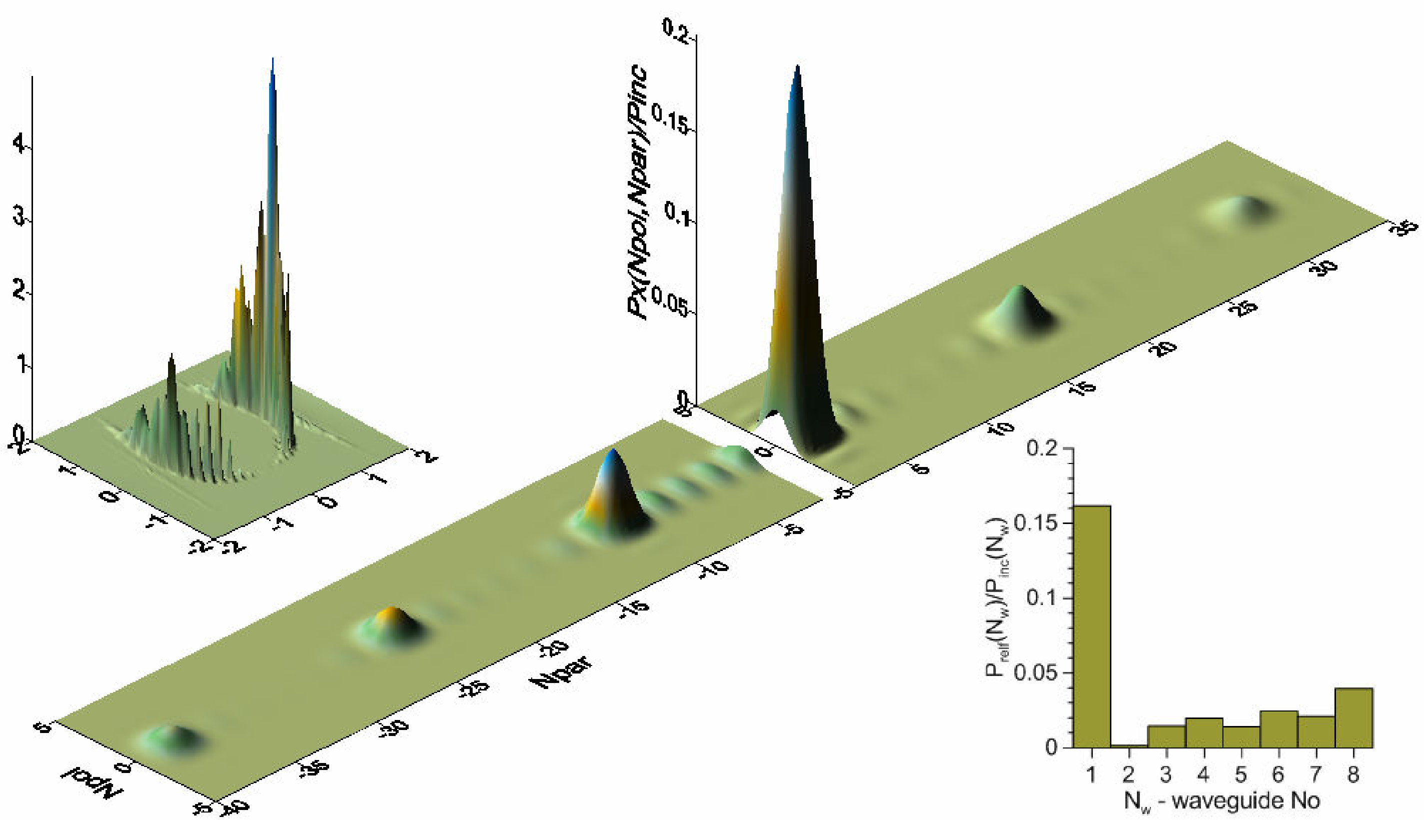
Two LH systems are considered for current drive and electron heating

1. Density limit ($\omega > 2\omega_{LH}$) restrict application of **original 1.3GHz 8 waveguide grill ($0.0148x0.165m^2$,0.002m septum)** for $n_0 < 2x10^{19} m^{-3}$. At phasing $\Delta\varphi = \pi/2$ then main peak has N_{par}=3.2 so accessibility limit is fulfilled in wide range of B_0
2. For NBI now installed at COMPASS $n_0 > 3x10^{19} m^{-3}$ is required. In collaboration with CEA Cadarache we prepare proposal of grill operating at 3.7GHz. Two rows of multijunction grills (2x6 or 2x8) are in consideration. The density limit for 3.7GHz is sufficiently high ($n_0 > 10^{20} m^{-3}$)



Spectral power density of the 1.3GHz grill - radiated spectrum for phasing $\pi/2$

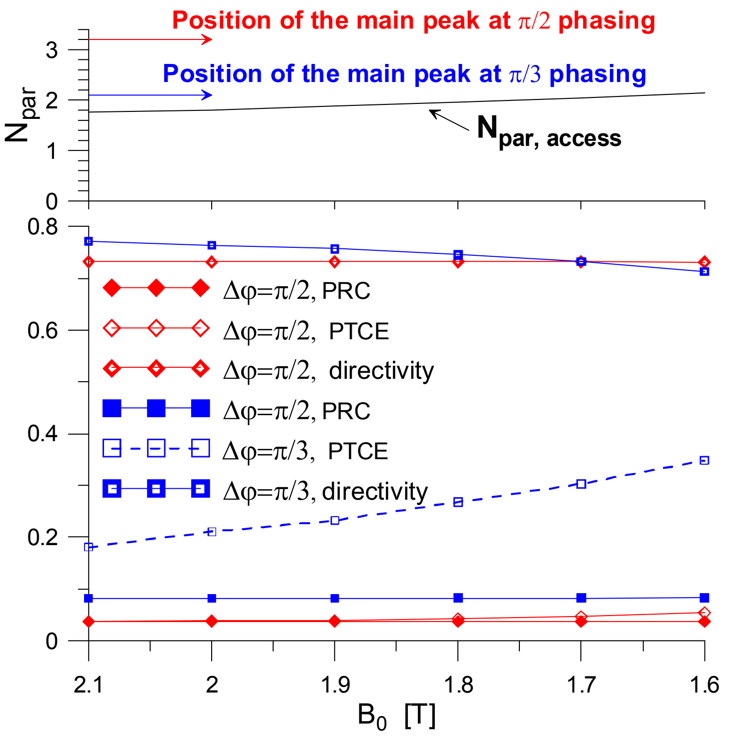
Parameters B_0=2.1T, $n_0=2x10^{19}m^{-3}$, $n_{surf}=2x10^{17}m^{-3}$
Power reflection coefficient 3.7%.Power transmission coefficient to the eigenmodes 3.7%. 92% is transmitted in accessible region as slow waves. Their directivity is 73%.



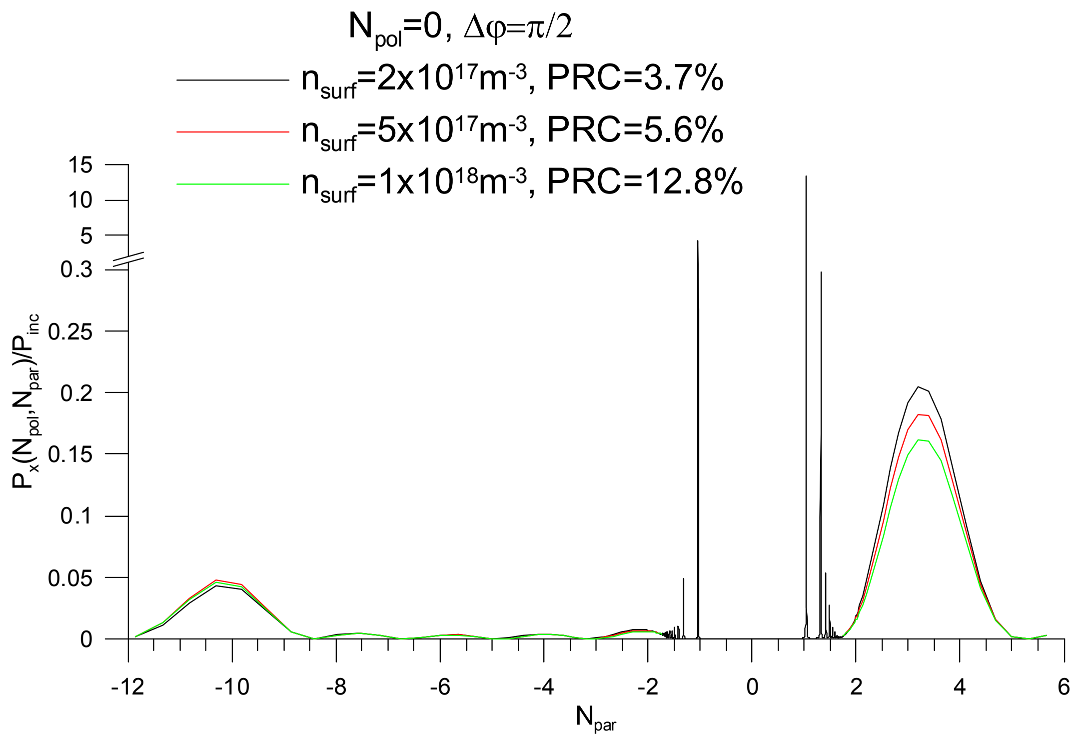
Optimum sensitivity to the change of plasma parameters

Power reflection coefficient (PRC), power transmission coefficient to eigenmodes (PTCE) and directivity of travelling waves in accessibility region.

Decrease of B_0

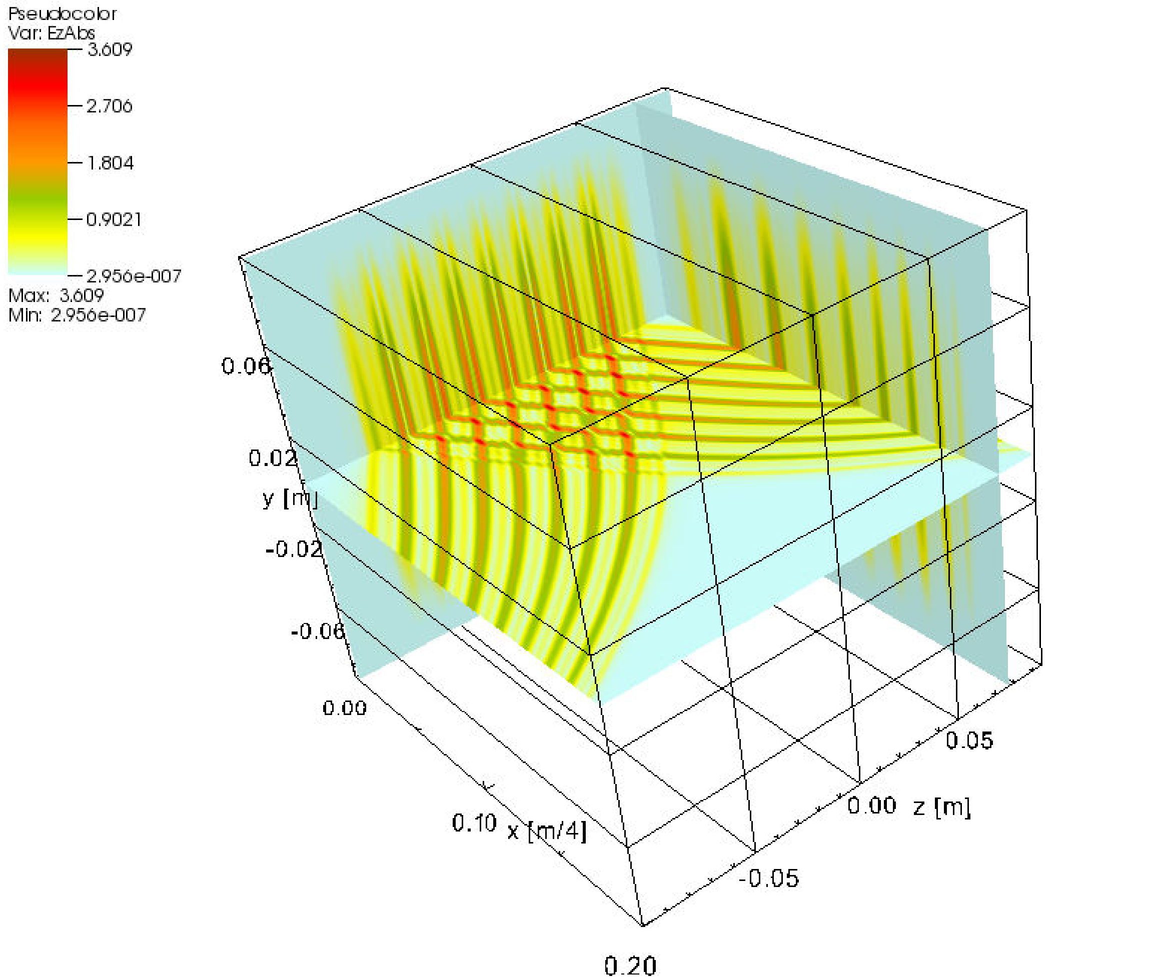


Increase of the plasma surface density



3D electric field launched by 1.3GHz grill

The electric field is normalized to the unit amplitudes of the incident waves in the waveguides

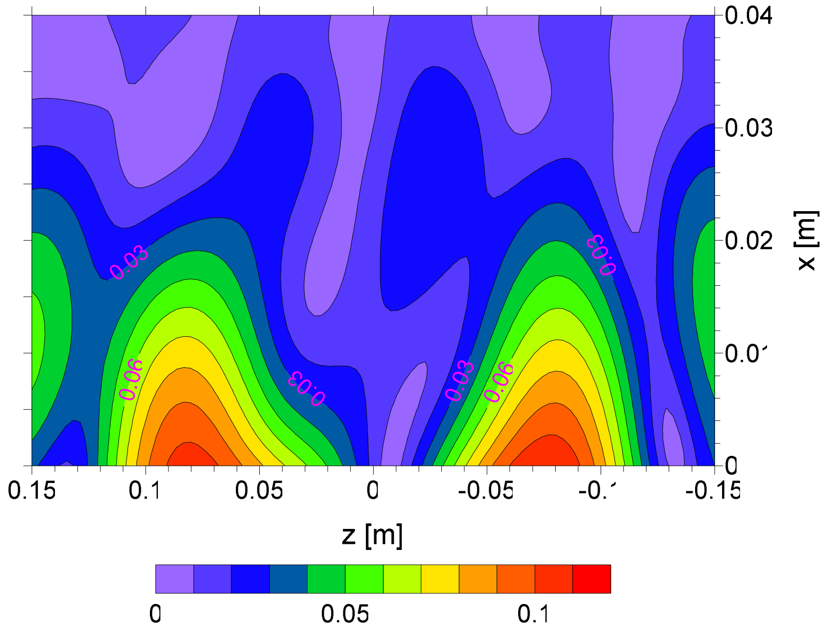


Contribution of eigenmodes to the electric field in front of grill

Product of mutual interference of eigenmodes is smooth and well confined both in the radial and poloidal directions. It consist from radially and poloidally standing waves of the size of the first eigenmode. In the toroidal direction it is a mixture of standing (main part) and propagating waves

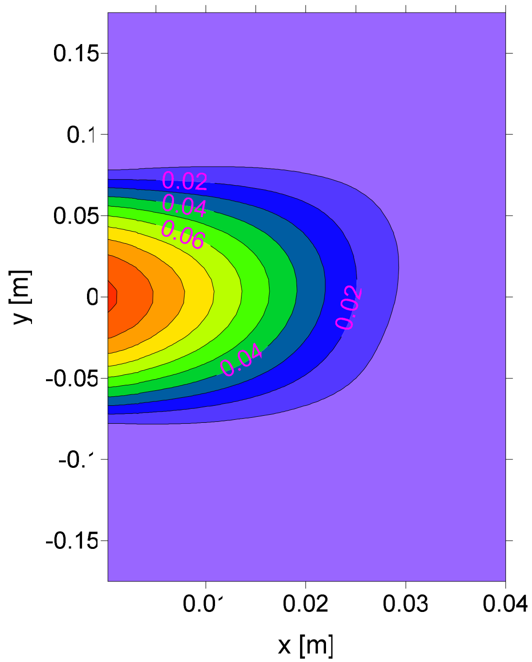
equatorial cross-section

$$\left|E_z^{\text{eigenmode}}{}_z(x, y=0, z)\right|$$



poloidal cross-section

$$\left|E_z^{\text{eigenmode}}{}_z(x, y, z=0.075\text{m})\right|$$



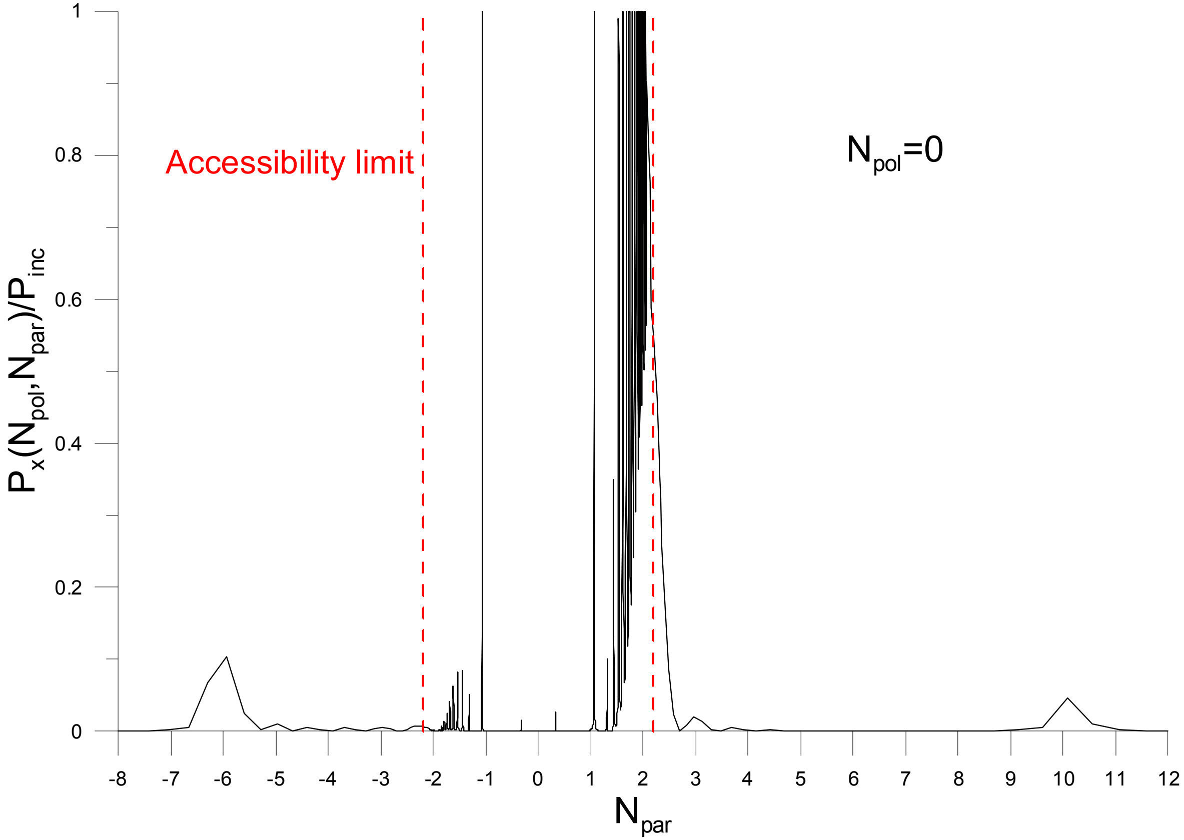
12 waveguide grill for COMPASS, f=3.7GHz

waveguides $0.009 \times 0.075 m^2$, septa 0.001m

$B_0 = 2.1T$, $n_0 = 3 \times 10^{19} m^{-3}$, $n_{surf} = 5 \times 10^{17} m^{-3}$

$\Delta\varphi = \pi/2$

Power reflection coefficient=5.5%, Power transmission coefficient to the eigenmodes= 59%,Directivity of waves in accessibility region = 55%, $N_{par,acess}=2.19$, $N_{par,peak}=2.0$. 60% power is lost, no directivity



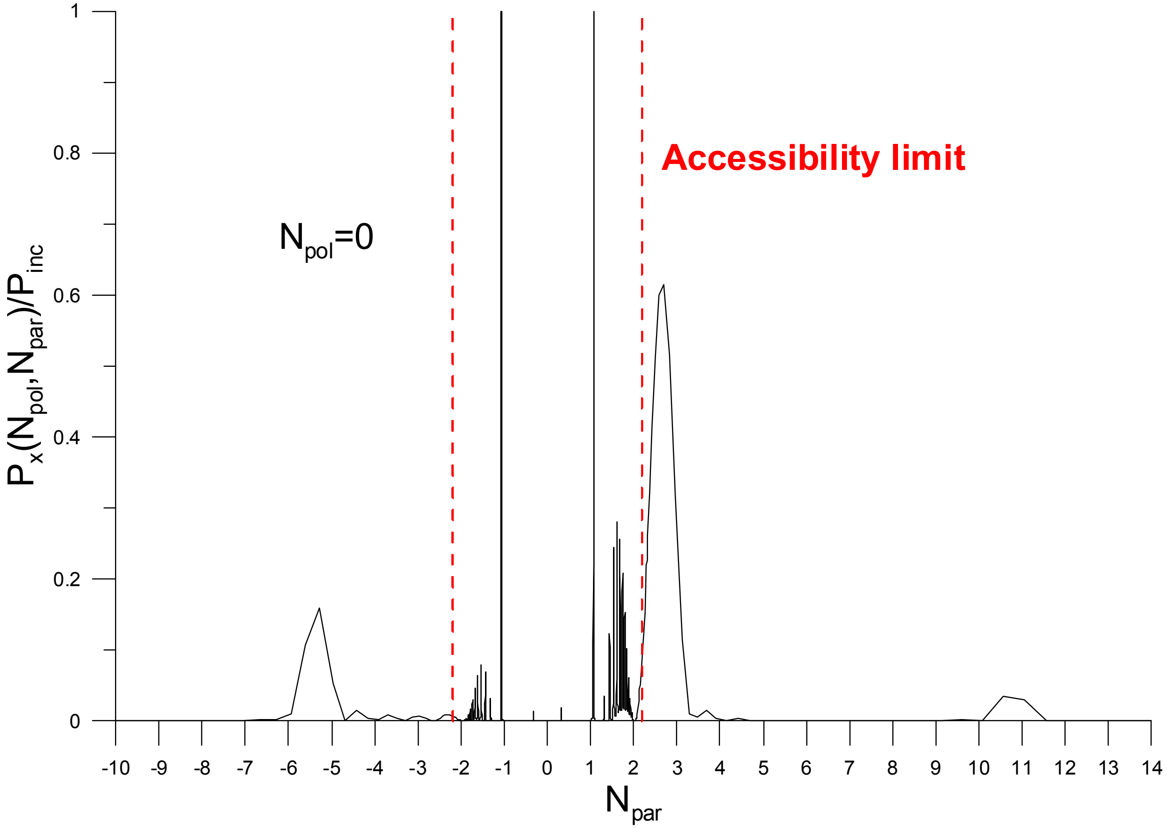
12 waveguide grill for COMPASS, f=3.7GHz

waveguides $0.009 \times 0.075 m^2$, septa 0.001m

$B_0=2.1T$, $n_0=3 \times 10^{19} m^{-3}$, $n_{surf}=5 \times 10^{17} m^{-3}$

$\Delta\varphi=2\pi/3$

Power reflection coefficient=14%, Power transmission coefficient to the eigenmodes= 6%, Directivity of waves in accessibility region = 73%, $N_{par,acess}$=2.19, $N_{par,peak}$=2.7. 20% power is lost, **Antenna can work in this situation**



Conlusions

- We have verified that the concept of one mesh of full wave solutions in k-space is sufficient for evaluation of all coupling admittances, spectral power density and its integrals. This mesh of solutions is independent of the grill parameters so it could be used for arbitrary structure.

- We have found the lines in the k-space where the plasma related parts of integrands of coupling admittances are singular and find how to cope with these problems.

- We are able to determine the electric field in front of the grill caused by the presence of eigenmodes in the inaccessible region.

- We have determined precisely which part of the radiated power by the waveguide grill goes into standing waves in the inaccessible region.

- We have shown that this part of the power is independent of the collisional frequency.

- Original LH 1.3GHz grill for COMPASS tokamak phased at $\Delta\varphi=\pi/2$ works well for densities below density limit in broad range of magnetic fields. Accessibility limit excludes phasing $\Delta\varphi=\pi/3$. Even for maximum B_0 substantial part of power is transmitted to plasma eigenmodes
- Proposed 12 waveguide grill at 3.7GHz with $\Delta\varphi=2\pi/3$ phasing can work in dense plasma

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Coupling of WEGA 2.45GHz 5 waveguide cylindrical LH grill

WEGA is a classical *l* = 2 stellarator with five field periods (*m* = 5), a major radius of *R* = 72 cm and a maximum effective plasma radius of *a*=11 cm, respectively. The toroidal magnetic field coils can be operated at 0.5 T for about 20 s

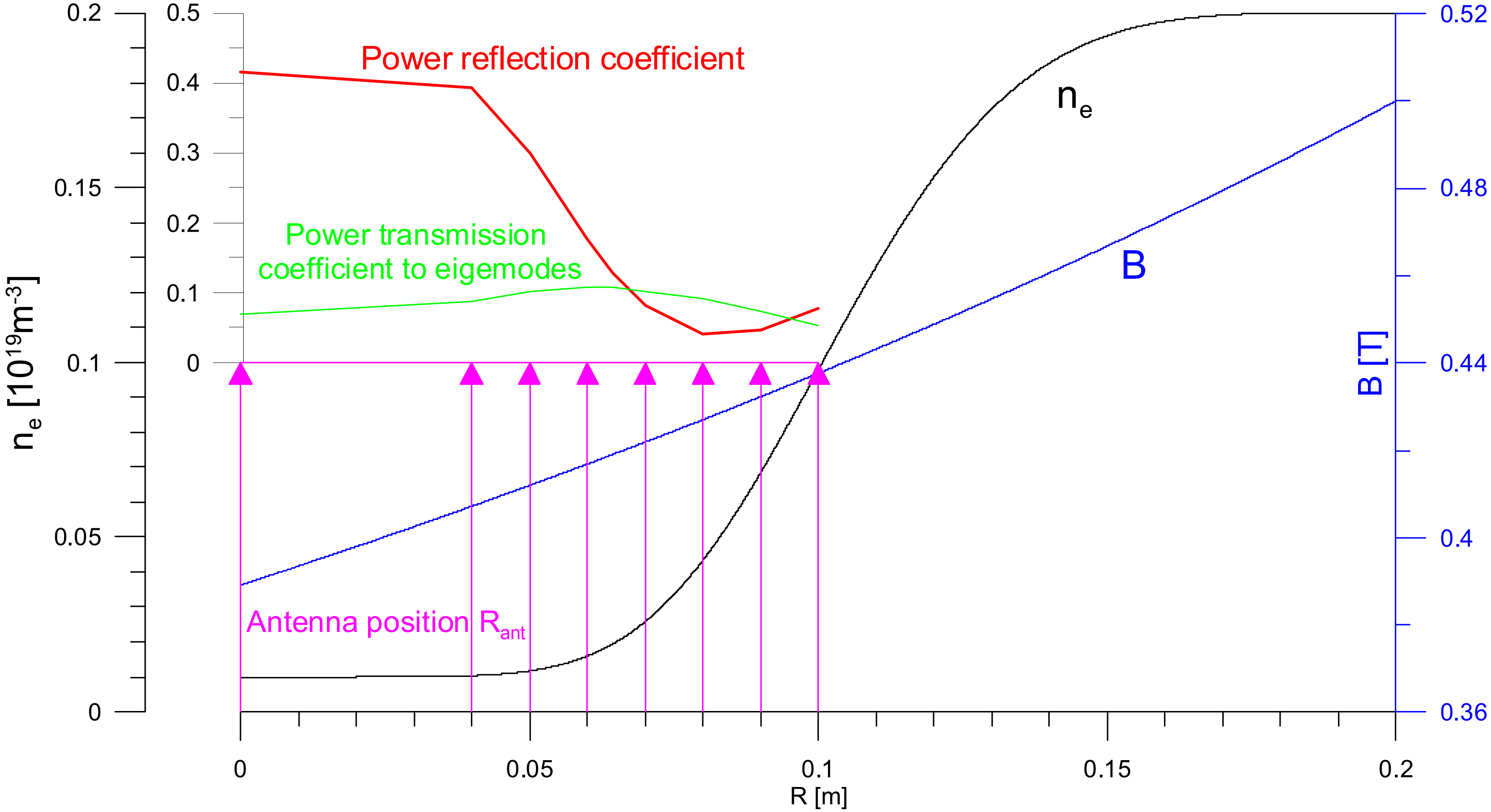
Grill has shape of cylinder with (∅ =85mm), 4 piecewise broken septa form 5 waveguide with phase shift $\Delta\varphi=\pi$, mouth shaped to fit the plasma surface. Heating of plasma is main purpose of this structure.

Simple rectangular 5 waveguide grill (16.28x85mm^2) was used for coupling tests

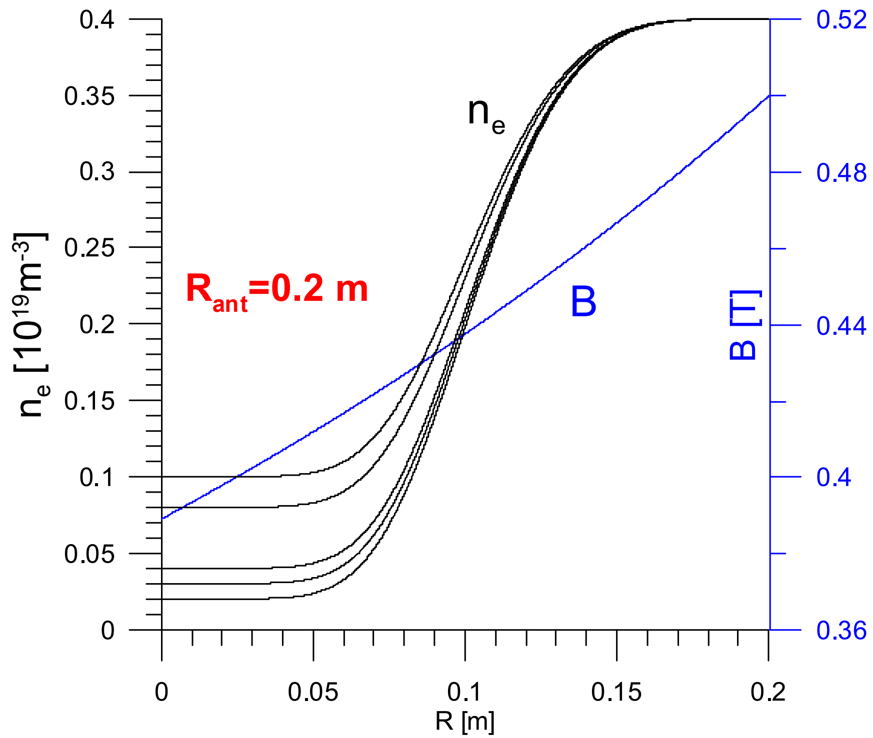
We tested the influence of the radial position of grill mouth, the influence of the plasma surface density and the density gradient at fixed plasma surface density, on the power reflection coefficient and the power transmission coefficient to eigenmodes.

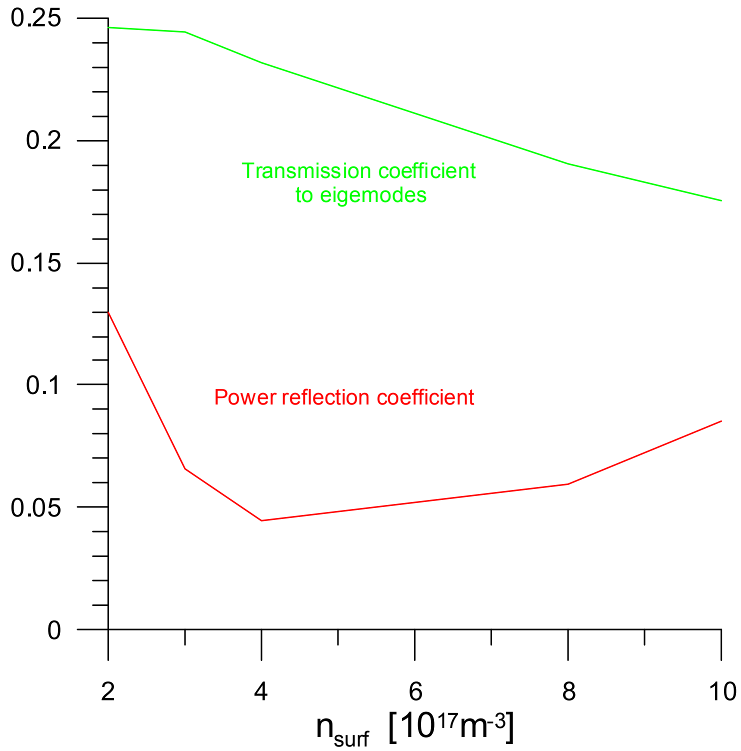
We find that our model predicts good coupling for plasma parameters considered what is contradictory with experiment.

Effect of antenna position



Effect of plasma surface density





Effect of plasma density gradient

$$n=(n_0-n_{surf})\exp\{[(R-R_{ant})/(R_{LCFS}F_B)]^{\wedge}P_{exp}\}+n_{surf}$$

R_{LCFS}=0.08m, F_B=1.33,