In low density plasmas of interest for magnetic fusion, the collision frequency is very small, so that a kinetic description is mandatory. However, when considering the one particle distribution function, one must address the physics in a 6 dimension phase space. To the latter complexity, one must also consider the high frequency Larmor motion of the particles.

A considerable simplification is achieved when reducing the phase space to 5D after assuming a time separation between the cyclotronic motion and the other time scales of interest for the physics. This introduces a new invariant of the motion that is equal to the magnetic moment at lowest order.

The scope of our work is to recast the analysis leading to the existence of this so-called adiabatic invariant into the framework of Hamiltonian formalism. One aspect of the work is to determine the 6D particle trajectories with appropriate separation between fast and slow motion.

The present effort addresses an alternative calculation of the expression of the adiabatic invariant, in particular in terms of an expansion that allows one to compute it with arbitrary precision. We study the motion of a particle (of unit charge) in an arbitrary static inhomogeneous external magnetic field B. And we build the normal form of this Hamiltonian system (with a non-canonical Poisson structure) exhibiting a non trivial constant of motion (its action variable, the "magnetic moment") which approximates the usual adiabatic invariant $p^2/2B$ where p is the momentum of the particle and B is the size of the magnetic field.

Our approach is intrinsic since it does not need to introduce an artificial point, the "guiding center", but only involves the position of the particle. Furthermore it does not require choosing an orthonormal frame aligned with the magnetic field, with its problem of gyro-gauge. We define an intrinsic gyro-phase and gyro-average operator. And we recover a posteriori, a guiding center and a gyro-radius (Larmor radius).